EBVM

The European Business Valuation Magazine

Business Valuation is a Question of Trust

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CEO FutureValue Group AG and honorary professor at the University of Dresden

**Professor Dr. Dr.
Dietmar Ernst**

Professor at the International School of Finance, Nürtingen-Geislingen University and director of the European Institute of Quantitative Finance.

The Simulation-Based Valuation of Companies and Their Strategies

– Classification, Methodology and Case Study –

Simulation-based business planning and business valuation are being increasingly used in business valuation practice. In contrast to CAPM-based DCF valuation, simulation-based DCF valuation derives the cost of capital from the risks that actually exist in the company. It can also consider market imperfections, insolvency risks and a varying degree of diversification of the valuation subject. When applying simulation-based business valuation, it is important for valuation practitioners to understand the basic ideas and valuation equations behind this approach. This article uses a simple example to convey all the essential aspects and steps of simulation-based business valuation.

Beta Estimation under Thin Trading Conditions

Estimates on betas may be distorted by thin (infrequent) trading effects, yielding incorrect estimates. Finance literature has proposed numerous techniques to eradicate the effects of thin trading, ranging from (il-)liquidity indicators indicating distortions in beta estimates to beta correction procedures directly correcting them in the traditional market model. This article provides an overview of comprehensive sets of 16 popular (il-)liquidity indicators and 10 popular beta correction procedures. Subsequently, these (il-) liquidity indicators and beta correction procedures are examined according to superiority in terms of accuracy (predictive ability) among themselves as well as against each other. The results indicate (i) the (il-)liquidity indicators to generally outperform the beta correction procedures in small as well as in large stock markets, across different levels of thin trading as well as across different levels of risk (beta magnitudes) and, (ii) the Illiquidity (Amihud-Hasbrouck) Indicator, the Return-to-Turnover Indicator as well as the Trade-to-Trade Method to dominate.

**Professor Dr.
Stefan O. Grbenic, StB, CVA**

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**Alexander Aronsohn,
FRICS**

IVSC Director of Technical Standards for Tangible Assets

Interview: Alexander Aronsohn on IVS Exposure Draft (2023)

IVSC has launched a three-month consultation on proposed updates to the International Valuation Standards (IVS). The consultation, which runs until 28 July 2023, is part of the IVSC's two-yearly cycle of standards updates. EBVM editors interviewed Alexander Aronsohn, who assists the IVSC Standards Review Board and its Asset Boards on drafting IVS, on key updates proposed in the Exposure Draft.

From the Editors

This issues' highlights

Dear Reader, as a member of the Editorial Committee, is it my pleasure to introduce to you the "summer 2023" issue of EBVM, the journal for the European business valuation profession that is published by EACVA and the IVSC. As you already know, the main purpose of EBVM is to promote valuation best practice through sharing knowledge among practitioners that operate in the many different cultures and professional environments in Europe. However, we also encourage our contributors to address more theoretical topics to provide new or more in-depth insights into the challenges of business valuation. A very welcome example of this kind is the article by *Stefan O. Grbenic* entitled Beta Estimation under Thin Trading Conditions. The Author, through extensive research, provides an overview of the application of 16 illiquidity indicators and of 10 correction procedures of beta values, examining them according to their superiority in terms of accuracy (predictive ability) among themselves as well as against each other.

In this issue you will find also an essay by *Werner Gleißner* and *Dietmar Ernst* that is dedicated to the The Simulation-Based Valuation of Companies and their Strategies. By discussing the estimate of cost of capital from the analysis of the risks that actually exist in the company the Authors offer a valuable insight in recent years' developments of s.c. "risk-adequate valuation", that tries to overcome traditional problems associated with derivation of cost of capital from the observation of capital markets returns (difficulty to adequately consider company specific risks, imperfection of markets, et coetera). The Authors classify two distinct approaches: the "risk coverage approach", that relies upon VAR-measures and considers financial restrictions, and the "risk-adequate valuation method" that derives cost of capital based on the coefficient of variation of earnings or cash flows. The article phocuses on the second approach whose practical application is illustrated by a case study.

This issue also features an interview with *Alexander Ahronson* on IVS Exposure Draft (2023), whose public consultation is, at the time I am writing, still pending. Among the various topics covered, I would like to highlight the discussion about the proposed IVS standards 104 (Data and Inputs) and 105 (Valuation Models), that have been conceived to address the increasing use of data and technology in valuation practice. Besides the Exposure Draft subject, the interview offers also a glimpse in the way IVS Boards will confrontate with the application of AI in the valuation process (the topic will be on the Agenda Consultation in 2024) in future. Finally, besides the beta and stock multiples data, this issue is enriched by new regional breakdown of the transaction (deal) multiples into Central and Western European and Southern Europe.

We hope you enjoy reading it and look forward to your feedback.

Ascanio Salvidio, Odcec, ACA, FRICS
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Imprint

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Publisher:
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European Association of Certified Valuators and Analysts
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Commercial Register: Frankfurt HRB 75029

ISSN 2940-8849

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Magazine typesetting:
Main-Post GmbH

Submission Dates:

Issue:	Submsion Dates:
Fall 2023:	15 August 2023
Winter 2023:	15 November 2023

The Simulation-Based Valuation of Companies and Their Strategies

– Classification, Methodology and Case Study –

Simulation-based business planning and business valuation are being increasingly used in business valuation practice. In contrast to CAPM-based DCF valuation, simulation-based DCF valuation derives the cost of capital from the risks that actually exist in the company. It can also consider market imperfections, insolvency risks and a varying degree of diversification of the valuation subject. When applying simulation-based business valuation, it is important for valuation practitioners to understand the basic ideas and valuation equations behind this approach. This article uses a simple example to convey all the essential aspects and steps of simulation-based business valuation.

Professor Dr. Werner Gleißner

CEO FutureValue Group AG and honorary professor at the University of Dresden. As chairman of the FutureValue Group AG, a research and development-oriented management consulting company, Werner Gleißner generally deals with value-based management based on enterprise valuation models for imperfect capital markets, taking account of corporate risk information and appropriate risk measures. His research and work both focus on risk management, rating and strategy development and the method development of risk aggregation and value-based management and also on investment and portfolio management. He published a large number of specialist publications and is lecturer at various universities.

Professor Dr. Dr. Dietmar Ernst

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I. Introduction and overview

The value of a company is a ratio that expresses, in condensed form, the (1) expected amount, (2) risk and (3) timing of cash flows generated by the company. In real, imperfect markets, this company value usually does not coincide with the stock market price (as price).[1] Due to the unrealistic assumptions and empirically unconvincing results, it is now obvious that discount rates (cost of capital rates) cannot be reliably determined using the Capital Asset Pricing Model (CAPM). The so-called build-up models[2] with a surcharge on the CAPM cost of capital also cannot be applied in a consistent valuation approach, because the assumptions underlying the CAPM mean that all valuation-relevant risks must be included in the beta factor.[3] Such build-up models are suitable as "price estimation models", i.e., use in for explaining prices observable on the market. They are conceptually unsuitable for determining a fundamental company value, and specifically for evaluating various options for action by a company.

Indeed, to determine a fundamental company value as a measure of the risk–return profile, it is necessary to capture the risks of the company itself (volatility of cash flows). It is unsuited, as with the beta factor, to considering the risks of fluctuations in stock returns. For a long-term-oriented investor, temporary stock return fluctuations are effectively irrelevant (see, e.g., the investment approach of Warren Buffet).

Particularly when evaluating a company's strategic options (e.g., two strategy variants or major investments), it is necessary to determine the potential different risks by means of risk analysis. These must then be considered in the valuation calculus (what–if analysis). In recent years, new valuation concepts have been developed to meet the challenges outlined here of valuing companies and their strategic options for action in a real, imperfect capital market with (credit) rating and financing constraints. The by the authors so-called semi-investment-theoretical valuation theory builds on the investment-theoretical valuation theory[4] (see the glossary for the most important terms) developed years ago, particularly in the German literature, and can be practically used following some simplifications. Semi-investment-theoretical valuation theory, when used as a method for risk-adequate valuation, does not assume perfect capital market. It considers rating and financing constraints and allows the derivation of discount rates based on an analysis of the risks of a company (or investment project). Historical stock return fluctuations of the valuation object (or data of a peer group) are not required. The derivation of the valuation equation and discount rates is based on only one, less restrictive assumption: two cash flows at the same time have the same value if they match in expected value and a chosen risk measure (such as standard deviation or value at risk).[5] Since companies display a large number of risks that are recorded in risk analyses, a so-called risk aggregation is required as a bridge between the risk analysis and the assessment. Based on the corporate planning and the analysis of existing opportunities and threats (risks) that trigger deviations from the plan, a large number of representative possible risk-related future scenarios are calculated using Monte Carlo simulation. By doing so, a realistic range of future cash flows is derived. From this, the expected value of cash flows (or flow-to-equity) and their levels of risk can be derived. From the volume of risk, a suitable risk-adjusted discount rate can again be derived: more risk leads to higher expected return requirements and correspondingly higher discount rates. Since in practice such a risk-adequate valuation always requires the use of a Monte Carlo simulation, this variant is referred to as a "simulation-based company valuation". In the practical implementation of simulation-based valuation, the familiar equations of the discounted cash flow (**DCF**) method can be used. It should be noted that expected cash flows and risk-adjusted discount rates are derived together and consistently with each other, taking into account the identified risks.

In this paper, Section 2 first explains semi-investment theory valuation, and the simulation-based business valuation that is based on it. This theory is related to traditional capital market-oriented (financing theoretical) valuation concepts, such as the discounted cash flow method based on the capital asset pricing model. In Section 3, the practical application of the method is illustrated using a case study. The starting point is the "traditional" valuation of a company based on the cost of capital derived using the CAPM. It is shown how the valuation is changed if, instead of (often ambitious) plan values, unbiased plan values are set that consider opportunities and risks. It is also shown how the risk volume of the cash flows can be used to derive a risk-adjusted discount rate that differs from the discount rate derived according to CAPM. Finally, it is made clear how the insolvency risk, i.e., the probability of insolvency expressed by the rating, can also be considered in the company valuation. In addition to the valuation of the company in its current situation, the valuation of a strate-

1 Ernst/Gleißner, Paradigm Shift in Finance: The Transformation of the Theory from Perfect to Imperfect Capital Markets Using the Example of Company Valuation, JRFM, vol. 15, no 9 (2022): 399-411; Shleifer/Vishny, The Limits of Arbitrage, The Journal of Finance, vol. 52, no. 1 (1997): 35-55 and Gromb/Vayanos, Limits of Arbitrage, Annual Review of Financial Economics, vol. 2, no. 1 (2010): 251-275.

2 See e.g. Damodaran, The Little Book of Valuation: How to Value a Company, Pick a Stock and Profit, 2011 and Grabowski, The size effect continues to be relevant when estimating the cost of capital, Business Valuation Review, vol. 37, no. 3 (2018): 93-109.

3 Kruschwitz/Löffler/Mandl, Damodarans Country Risk Premium – und was davon zu halten ist, WPg, no. 4 (2011): 167-176 and Ernst/Gleißner, Damodarans Länderrisikoprämie, WPg, no. 23 (2012): 1252-1264.

4 Matschke, Funktionale Unternehmensbewertung, Band II, Der Arbitriumwert der Unternehmung, 1979; Hering, Unternehmensbewertung, 4th ed. 2021 and Matschke/Brösel, Business Valuation, 2021.

5 Dorfleitner/Gleißner, Valuing streams of risky cashflows with risk-value models, Journal of Risk, vol. 20, no. 3 (2018): 1-27.

Table 1: Comparison of valuation methods

	Risk-adequate valuation ("incomplete replication")		Capital market-oriented valuation	
	Risk coverage approach	Coefficient of variation approach	CAPM (return equation)	Multi-factor models*
Level of information	Individual level of information		Information processed on the capital market	
Risk reference	Earnings (EBIT or cash flow) - historical or - future-related (Monte Carlo simulation)		As a rule, stock returns (for derivation beta)	Stock returns
Risk effect on	Discount rate and at the same time expected earnings (cash flow)		Discount rate (expected earnings are not linked)	
Risk measures	Value at risk due to financing restrictions	Standard deviation	Standard deviation (by beta factor)	Several or not explicit
Alternative investment	- risk-free investment - risky stock index (Note: other investments possible)		- risk-free investment - (theoretical) market portfolio	Not explicit
Diversification (valuation subject)	Any / individual (often d = 1)	Any / individual (often d = 1, d = 0.5, d = *p*)	Perfectly diversified (d = *p*) *p* is the correlation to the market return required to determine the beta factor of the CAPM (see Chapter III)	Not explicit

*See e.g. Fama/French, A five-factor asset pricing modell, Journal of Financial Economics, vol. 116, no. 1 (2015): 1-22.

gic option for action, an efficiency enhancement program, is also carried out. In this way, the risk-return profile of alternative options for action can be evaluated in a well-founded manner, which is necessary to developing comprehensible and well-founded decision-making proposals for upcoming entrepreneurial decisions (Business Judgement Rule).[6] A concise conclusion ends the article.

II. Valuation methods in comparison

Capital market-oriented valuation approaches, such as the CAPM, derive risks from capital market data. The calculations are based on fluctuations in stock returns. The CAPM has the disadvantage that company-specific risks, which arise from the company's risk analysis, are not adequately considered. In the beta factor of the CAPM, historical stock return fluctuations are evaluated (and the risks of a company's future cash flows are not explicitly considered). Given the condition of imperfect capital markets, valuations based on historical capital market data are problematic.[7] As explained in the introduction, a foundation for "risk-adequate valuation" has been developed in recent years in the form of semi-investment-theoretical valuation theory. This valuation method directly uses the result of the analysis of the future risks of a company to determine the discount rate and thus the value of the company. As an alternative to DCF valuation based on CAPM, two variants of "risk-adequate valuation" have developed, whose valuation equations can each be derived using the "incomplete replication" method.[8]

1. The risk coverage approach, which uses the value at risk measure and considers financing restrictions. This approach is only classified here (see Table 1) and not explained in more detail.[9]
2. The risk-adequate valuation method, which derives the costs of capital via the coefficient of variation of earnings or cash flows.[10]

Table 1 compares the two approaches of risk-adequate (semi-investment theory) valuation with the approaches of capital market-oriented valuation.[11] In the case study in Chapter III, a risk-adequate valuation is carried out and compared with a valuation based on CAPM. In the risk-adequate valuation, the discount rate is derived from the coefficient of variation of the cash flows (flow-to-equity)

6 Gleißner, Entrepreneurial Decisions, Entrepreneurial Decisions – Avoiding liability risks (Section 93 AktG, Business Judgement Rule), Controller Magazin, vol. 45, no. 1 (2021): 16-21.
7 See for criticism Dempsey, The Capital Asset Pricing Model (CAPM): The History of a Failed Revolutionary Idea in Finance?, Abacus, vol. 49, no. S1 (2013): 7-23 and Dempsey, The CAPM: A Case of Elegance is for Tailors?, Abacus, vol. 49, no. S1 (2013): 82-87; Rossi, The Capital Asset Pricing Model: A Critical Literature Review, GBER, vol. 18, no. 5 (2016): 604-617; Schildbach, Modigliani/Miller-Thesen und CAPM: Irrlehren statt wegweisender Theorien, BFuP, no. 4 (2022): 375 et sqq. and Fernández, Is It Ethical to Teach That Beta and CAPM Explain Something?, working paper, 2019, SSRN-ID 2980847 (last access 05.05.2023).
8 Gleißner, Grundlagen des Risikomanagements. Handbuch für ein Management unter Unsicherheit, 4th ed. 2022: 490-493.
9 See for a more detailed explanation Ernst/Gleißner, Total Beta: A View from Outside, The Value Examiner (to appear 2023).
10 Gleißner, Cost of capital and probability of default in value-based risk management, Management Research Review, vol. 42, no. 11 (2019): 1243-1258.
11 See further Gleißner/Meckl, Methoden der Unternehmensbewertung und ihre Anwendung bei M&A, WiSt (to appear 2023).

("coefficient of variation approach"). The coefficient of variation can be determined via risk analysis and risk aggregation and is formally defined as the standard deviation in relation to the expected value of the cash flows.

The method of risk-adequate valuation is always based on information about the risks of the company itself, which are determined by means of risk analysis. In principle, it is possible to derive the risk measures required for this purpose from historical fluctuations in earnings or cash flows. Statistical use is made of fluctuations in earnings and the results to determine, for example, the corresponding coefficient of variation of profits.[12] In principle, it is preferable to use a forward-looking valuation, which looks at the risks that are decisive for the value and viability of a company. In such cases, the basis yields an analysis of future risks and risk aggregation. If the valuation of a company or its strategic options for action is based on a risk analysis and a Monte Carlo simulation for risk aggregation, we arrive at a "simulation-based valuation". The central characteristics of a simulation-based valuation are as follows:[13]

1. Considering the effect of corporate risks on integrated planning;
2. Using of Monte Carlo simulation for risk aggregation.

Initially, a simulation-based valuation does not imply commitment to a specific valuation theoretical framework. The use of that valuation method is possible in

1. An investment theoretical valuation;[14]
2. A semi-investment-theoretical valuation by means of "imperfect replication";[15]
3. In a finance-theoretical valuation based on the CAPM.[16]

The frequency distributions of the cash flows from the simulation are each condensed to the expected value in the valuation. The risk of the cash flows is expressed by a risk measure, such as standard deviation or value at risk.[17] With a risk-value model and the imperfect replication method[18], the risk-adequate present value can be calculated, taking into account the (a) amount, (b) risk and (c) timing of cash flows. The value calculated in this way only represents a certain amount of money equivalent to an uncertain future cash flow with the same expected value and risk. Neither capital market data on the valuation object nor the hypothesis of perfect capital markets are required for the valuation. Besides assumptions about alternative investment options, e.g., government bonds with AAA-rating and a world equity portfolio, we only need one other assumption: two cash payments at the same time coincide exactly in value if they have the same expected value and the same values of the chosen risk measure. Thus, the risk-adequate cost of capital can be derived[19] without historical capital market data (beta factor of the company or peer group).

The defining characteristic of a simulation-based assessment is the explicit consideration of business risks (opportunities and threats) and the application of Monte Carlo simulation for the calculation of risk-related future scenarios. The resulting "multi-value" planning (bandwidth planning) structure allows expected values of cash flows or earnings to be derived directly, determines and captures insolvency risk, and allows discount rates to be derived directly from the uncertainty of cash flows (i.e., without evaluating stock return fluctuations). With a simulation-based assessment, the new legal requirements for risk management are also met (e.g., in Germany § 1 StaRUG).

The main characteristics and advantages of a "simulation-based business valuation" based on the analysis of business risks can thus be summarized as follows:[20]

1. Only with simulation-based planning can the expected values of cash flows or earnings be derived in a comprehensible manner.
2. A plausibility check of the planning and planning logic is carried out.
3. A simulation-based valuation can be used to consider the impact of insolvency risk on the value of the company.
4. A simulation-based business valuation allows the derivation of a risk-adjusted discount rate (cost of capital) directly from the simulation results.
5. Simulation-based valuation can represent a basis for preparing business decisions because planned future changes in planned values and risks can be considered.

12 Gleißner, Unternehmenswert, Ertragsrisiko, Kapitalkosten und fundamentales Beta – Studie zu DAX und MDAX, BewertungsPraktiker, no. 2 (2016): 60-70.
13 See Gleißner, Simulationsbasierte Unternehmensbewertung: Methode und Nutzen, BewertungsPraktiker, no. 3 (2021): 84-87.
14 Hering/Schneider/Toll, Simulative Unternehmensbewertung, BFuP, vol. 65, no. 3 (2013): 256-280.
15 Gleißner, Risikoanalyse und Replikation für Unternehmensbewertung und wertorientierte Unternehmenssteuerung, WiSt, no. 7 (2011): 345-352; Gleißner, op. cit. (footnote No. 10): 1243-1258; Dorfleitner/Gleißner, op. cit. (footnote No. 5): 1-27; Dorfleitner, On the use of the terminal-value approach in risk-value models, Annals of Operations Research, vol. 313, no. 2 (2020): 877-897; Ernst, Simulation-Based Business Valuation: Methodical Implementation in the Valuation Practice, JRFM, vol. 15, no. 5 (2022): 1-17.
16 On the use of the less common certainty-equivalent variant of the CAPM, in which the risk of the cash flows is included in the valuation calculation, see Robichek/Myers, Conceptual problems in the use of risk-adjusted discount rates, The Journal of Finance, vol. 21, no. 4 (1966): 727-730 and Rubinstein, The Fundamental Theorem of Parameter Preference security valuation, JFQA, vol. 8, no. 1 (1973): 61-69.
17 Ernst, Risk Measures in Simulation-Based Business Valuation: Classification of Risk Measures in Risk Axiom Systems and Application in Valuation Practice, Risks, vol. 11, no. 1 (2023): 1-13.
18 Gleißner, op. cit. (footnote No. 15): 345-352; Gleißner, op. cit. (footnote No. 10): 1243-1258 and Dorfleitner/Gleißner, op. cit. (footnote No. 5): 1-27.
19 Gleißner, op. cit. (footnote No. 10): 1243-1258.
20 According to Gleißner, op. cit. (footnote No. 13): 84-87 and Ernst, op. cit. (footnote No. 15): 1-17.

6. Finally, there is an additional benefit for companies, namely, compliance with legal requirements for risk management.[21]

In the following case study, a company is first valued on the basis of the CAPM, with the exception of insolvency risks. Step by step, the valuation is then improved, considering the company's earnings and insolvency risks. This shows how a sound assessment of earnings and insolvency risks is possible based on risk analysis and risk aggregation.

III. Case study: Company valuation and strategy evaluation

1. Overview

An important field of application of risk analysis in conjunction with adequate risk valuation[22] is strategy evaluation.[23] This serves to prepare decisions to be made by the board of directors or management. In the following case study, a strategy evaluation is carried out for a listed company. This is based on the standard deviation or the coefficient of variation of cash flows, which captures the extent of possible deviations from the plan ("output-oriented valuation"). As in the CAPM, the shareholders' risk diversification options are also taken into account. The effect of an outsourcing strategy on the company value is to be examined.[24] In doing so, it will be determined whether this strategy makes sense under consideration of return and risk. The idea of outsourcing was derived from the consideration of the strategic positioning and the essential success potentials. The relevant section of the value chain does not show any viable potential for success in the company. Thus, an outsourcing strategy was developed in cooperation between the departments established for controlling, production and logistics, in order to:

- Reduce costs (and increase earnings) by lowering purchasing prices;
- At the same time, to reduce risks by replacing part of the fixed costs with variable costs.

Whether the corresponding concept is promising and thus leads to an increase in value is examined in the case study.

In preparation for the strategy evaluation, the aggregated total risk volume (earnings risk), the rating and the risk-adjusted company value (as a measure of success) are first determined to establish the status quo. Subsequently, an evaluation is performed of how a planned measure set out to optimize the supply chain (outsourcing of a key section of the value chain) will affect these parameters. The aim is to use the risk-return profile (and the enterprise value as a performance measure) to make a well-founded assessment of the economic added value of this idea, which initially appears strategically plausible.

2. The company valuation with CAPM

The initial situation of the company can be characterized by the following figures: With total assets representing the capital employed ($\mathbf{CE}$) of CE = € 100 million, the company has an equity ratio of 30%. The interest-bearing net financial liabilities (debt) amount to D = € 50 million, the non-interest-bearing debt to € 20 million. In the fiscal year in t(0), an operating profit ($\mathbf{EBIT}$) of € 11.5 million and a profit (taxes are neglected for the sake of simplicity) of € 10 million were generated on sales of € 200 million. Without growth, the entire profit can be distributed. The valuation is carried out using the flow-to-equity method (capitalized earnings method), but the entity variant, based on FCF, is shown for comparison[25]. The difference between EBIT and profit is the interest expense. The return on capital employed ($\mathbf{ROCE}$) is therefore as follows:

$$\mathrm{ROCE}=\frac{\mathrm{EBIT}}{\mathrm{CE}}=\frac{11.5}{100}=11.5\% \qquad (1)$$

In corporate planning, for the financial year $\mathbf{t(1)}$ and all subsequent years, € 10 million profit is assumed with the highest probability (as planned value $\mathbf{Profit^{plan}}$). This profit is to be distributed to the owners (earnings = flow to equity = profit). Due to the difficult market conditions, management does not expect any growth in the future (growth rate $\mathbf{g = 0}$). The valuation-relevant free cash flow ($\mathbf{FCF}$) also allows the full distribution of profits. Using the Gordon-Shapiro model for an infinite annuity, the company value ($\mathbf{Value}$) is calculated here – based on assumptions considered to be credible and plausible – as follows (growth rate $\mathbf{g}$ would reduce $\mathbf{FCF}$ by $\mathbf{CE \cdot g}$ and earnings by $\mathbf{CE \cdot g \cdot equity\ ratio}$):

$$\mathrm{Value}_1=\frac{\mathrm{FCF}^{\mathrm{plan}}}{\mathrm{WACC}-\mathrm{g}}-\mathrm{D}\approx\frac{\mathrm{Profit}^{\mathrm{plan}}}{\mathrm{c}}=\frac{\mathrm{Earning}^{\mathrm{plan}}}{\mathrm{c}} \qquad (2)$$

The discount rate (cost of equity, $\mathbf{c}$) is traditionally first derived based on historical stock return fluctuations using CAPM. Assuming, for comparison purposes, an expected return on the market portfolio ($\mathbf{r_m^e}$) of 8%, a risk-free rate ($\mathbf{r_f}$) of 3% and a standard deviation of the market return (σ_m) of 20%, the beta factor can first be determined if the following two pieces of information are also derived from the historical stock price fluctuations (capital market data):

21 In close accordance with Gleißner, op. cit. (footnote No. 8): 490-493 and 513-523.

22 Semi-investment theory valuation based on a risk-value model see Dorfleitner/Gleißner, op. cit. (footnote No. 5): 1-27 and Gleißner, op. cit. (footnote No. 10): 1243-1258.

23 See Gleißner/Ernst, Company valuation as result of risk analysis: replication approach as an alternative to the CAPM, Business Valuation OIV Journal, vol. 1, no. 1 (2019): 3-18 with an alternative case study.

24 According to Gleißner, Die risikogerechte Bewertung alternativer Unternehmensstrategien: ein Fallbeispiel jenseits CAPM, BewertungsPraktiker, no. 3 (2013): 82-89 and Gleißner, op. cit. (footnote No. 8): 433-434.

25 Matschke/Brösel, op. cit. (footnote No. 4).

- Correlation (ρ) of the stock return to the market return 0.5;
- Standard deviation of the stock return (σ_i) 25%.

The beta factor is calculated as follows:

$$\beta = \rho \cdot \frac{\sigma_i}{\sigma_m} = 0.5 \cdot \frac{0.25}{0.2} = 0.625 \qquad (3)$$

In accordance with the well-known CAPM return equation, the discount rate, assuming that the CAPM assumptions are valid, is as follows,

$$r_e = c^{CAPM} = r_f + \left(r_e^m - r_f\right) \cdot \beta = = 0.03 + \left(0.08 - 0.03\right) \cdot 0.625 = 6.1\% \qquad (4)$$

and for the company value,

$$\text{Value}_1 = \frac{\text{Earning}^{plan}}{c^{CAPM}} = \frac{10}{6.1\%} = 163.9 \qquad (5)$$

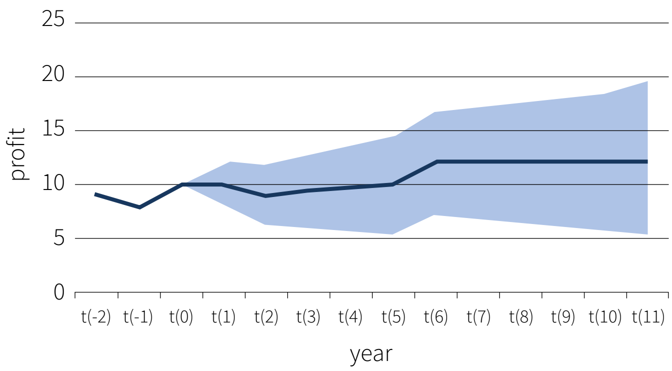
Value1 stands for the value of variant 1, which corresponds to the CAPM approach. In this "traditional" approach, information on the risks of future earnings is not considered, nor is the probability of insolvency (p) expressed by the rating. Furthermore, no consideration is given to the extent to which the "planned value", in this case the most probable value (modal value), is actually unbiased.

3. Risk-adequate company valuation in the initial situation

In the following, the valuation case is refined to determine the "risk-adequate value" in the initial situation. In doing so, it is assumed that a quantitative risk analysis has been carried out as part of risk management, and that the aggregated total risk volume has been calculated using Monte Carlo simulation, which is explained in more detail below.

In our case study, the risk aggregation for the status quo of the company (i.e., before implementation of the planned measure (outsourcing)) results in the following situation: The original planned value of the profit of € 10 million is not "unbiased". This value does not show what profit can be expected "on average" across all risk-related possible scenarios.[26] We can easily derive the expected value of the profit as an average of all simulated scenarios from the risk aggregation. It amounts to € 9 million (the individual risks are not presented here). This means that an average of € 9 million can be expected for all risk-related possible future scenarios (as mentioned, this value is considered representative of the future below). Of course, it is also possible to look at the earnings of the detailed planning period independently.[27] Due to an existing risk overhang compared to the opportunities, the expected value relevant to valuation is therefore lower than the planned value of € 10 million (see Figure 1).

Figure 1: Range of profit from Monte Carlo simulation (risk aggregation)



The expected return on assets, which strongly influences the rating in addition to the equity ratio (30%), is calculated as follows, assuming the time-variant expected value of profit (€ 9 million)[28], but the case with consideration of the simulation result) and interest (€ 1.5 million) (EBIT as € 10.5 million).

$$\text{ROCE} = \frac{10.5}{100} = 10.5\% \qquad (6)$$

Adequate consideration of the probability of insolvency p (of the rating), and the impacts of opportunities and threats (risks) relevant to the expected value of earnings or cash flows, is necessary in any proper company valuation, especially in strategy valuation. The probability of insolvency acts like a "negative growth rate" in the long term, that means:

$$\text{Value} = \frac{\text{Earning}^e\left(1 - p\right)}{c + p} \qquad (7)$$

It is important to note that the insolvency probability is not a premium on the cost of capital (as in the build-up models). There is no double counting of a risk because p "only" captures the effect on expected earnings over time (just like a growth rate).

Given a growth rate g[29], the (conditional) expected values of the earnings Earning^e (without insolvency – conditional ex-

26 Gleißner, op. cit. (footnote No. 10): 1243-1258 and Gleißner, op. cit. (footnote No. 8): 318-325, especially also for simulation-based risk aggregation.

27 Cf. Gleißner/Ernst, op. cit. (footnote No. 23): 3-18.

28 Still without considering failures; index 2 doesn't mark here period 2 (t=2).

29 For the relationship between w and k in the case of inflation-, retention- and tax-indexed (endogenous) growth, see Tschöpel/Wiese/Willershausen, Unternehmensbewertung und Wachstum bei Inflation, persönlicher Besteuerung und Verschuldung (Teil 1 und 2), WPg, no 7 (2010): 349-357 and WPg, no 8 (2010): 405-412.

pected value – as well as period-invariant insolvency probability, here for T, i.e. after detailed planning phase) and a discount rate c, the following equation results for the **Value** in the going concern phase (terminal value) as a function of the insolvency probability p (i.e. after detailed planning phase) and a discount rate c results in Equation (8) for the enterprise value in the going concern phase (terminal value) depending on the insolvency probability p[30]:

$$\text{Value} = \sum_{t=1}^{\infty} \frac{\text{earning}^e \cdot (1-p)^t \cdot (1+g)^t}{(1+c)^t} = \\ = \frac{\text{earning}^e \cdot (1-p) \cdot (1+g)}{c-g+p \cdot (1+g)} \qquad (8)$$

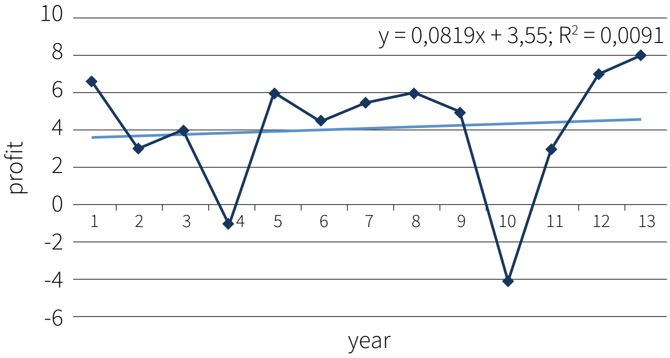
This also applies if one wishes to derive the cost of capital (discount rates) on the basis of the CAPM.

So additionally, the rating is also considered (risk of insolvency/bankruptcy). This indicates the insolvency risk, which is expressed by the probability of insolvency. Rating and insolvency probability p can be estimated using the Monte Carlo simulation.[31] In a simplified form, the probability of insolvency p can be estimated using financial ratios for the planned year equity ratio and **ROCE**, by means of the following "mini-rating"[32]:

$$p = \frac{0.265}{1+e^{-0.41+7.42 \cdot \text{equity ratio}+11.2 \cdot \text{ROCE}}} = \\ = \frac{0.265}{1+e^{-0.41+7.42 \cdot 0.30+11.2 \cdot 0.105}} = 1.3\% \qquad (9)$$

The insolvency probability derived from a simple financial ratio system can be estimated even more soundly using somewhat more complex ratio systems. For a supplementary plausibility check of the insolvency probability, considering the aggregated earnings risks neglected in financial ratios, risk aggregation can again be used. In this process, each simulation runs checks of whether illiquidity or (less relevant) over-indebtedness occurs. Often, the company can already be assumed to be illiquid if either (a) covenants are breached in a simulation run, and/or (b) a financial ratio rating of "B" is no longer guaranteed due to losses. In our case study, the risk simulation results in a fairly similar probability of insolvency. In the following, we will continue to calculate with using insolvency probability of $p = 1.3\%$ given in Equation (9).

Figure 2: Profit development in recent years (in € million)*



*Gleißner, op. cit. (footnote No. 4), pp. 82–89.

If the company is to be assessed from the perspective of a long-term committed investor (owner) and it is assumed that the valuation-relevant risks of future earnings are not reflected in historical stock returns, the following derivation of the cost of capital rates based on earnings risks is recommended.

It is important to emphasize here that the risk-adequate valuation method can always be applied if the risk content of the cash flows or earnings (flow-to-equity) is captured by a risk measure, such as σ_{Earning} here. In the simplest case, the determination of the risk measure can simply be performed as an estimate or be based on a statistical evaluation of historical profit fluctuations.[33]

However, it is recommended to use the best available information about the future risks of a company, which ultimately determines the level of the risk measure, i.e., the standard deviation. This is made possible using the simulation-based variant of the risk-adequate valuation of the company outlined here. The starting point here, as explained in section 2, is a risk analysis plus risk aggregation using Monte Carlo simulation. In this procedure, the main risks of a company are first systematically identified.[34] For example, all of a com-

30 Franken/Gleißner/Schulte, Insolvenzrisiko und Berücksichtigung des Verschuldungsgrads bei der Bewertung von Unternehmen – Stand der Diskussion nach Veröffentlichung des IDW Praxishinweises 2/2018, Corporate Finance, no. 3-4 (2020): 84-96; Gleißner, op. cit. (footnote No. 10): 1243-1258; Knabe, Die Berücksichtigung von Insolvenzrisiken in der Unternehmensbewertung, 2012 and Saha/Malkiel, DCF Valuation with Cash Flow Cessation Risk, JAF, vol. 22, no. 1 (2012): 175-185.

31 See Gleißner, op. cit. (footnote No. 10): 1243-1258 on simulation-based rating and evaluation procedures; for example using the "strategy navigator" software used here.

32 See Gleißner, op. cit. (footnote No. 8): 433-434 and alternative Altman, Predicting financial distress of companies: revisiting the Z-score and ZETA models, working paper of New York University, 2000, http://pages.stern.nyu.edu/~ealtman/Zscores.pdf (last access 05.05.2023) or Drobetz/Heller, What Factors Drive Corporate Credit Ratings? Evidence from German SMEs and Large Corporates, Working Paper Series, 2014, SSRN-ID 2392377 (last access 05.05.2023) and Krotter/Schüler, Empirische Ermittlung von Eigen-, Fremd- und Gesamtkapitalkosten: eine Untersuchung deutscher börsennotierter Aktiengesellschaften, zfbf, vol. 65 (2013): 390-433, which approximate S&P ratings using a simple financial ratio system.

33 See Figure 2 and Gleißner/Günther/Walkshäusl, Financial sustainability: measurement and empirical evidence, JBE, vol. 92, no. 3 (2022): 467-516.

34 Gleißner, op. cit. (footnote No. 10): 1243-1258 and Hunziker, Enterprise Risk Management: Modern Approaches to Balancing Risk and Reward, 2019.

Cost of capital in real time

Researching and preparing the data for the derivation of cost of capital or multiples does not have to be an elaborate process. The KPMG Valuation Data Source calculates the WACC and multiples at the push of a button. The tool groups together all important cost of capital parameters, including country risk premiums, credit spreads, sector- and peer-group-specific beta factors as well as multiples – updated monthly in an interactive dashboard.

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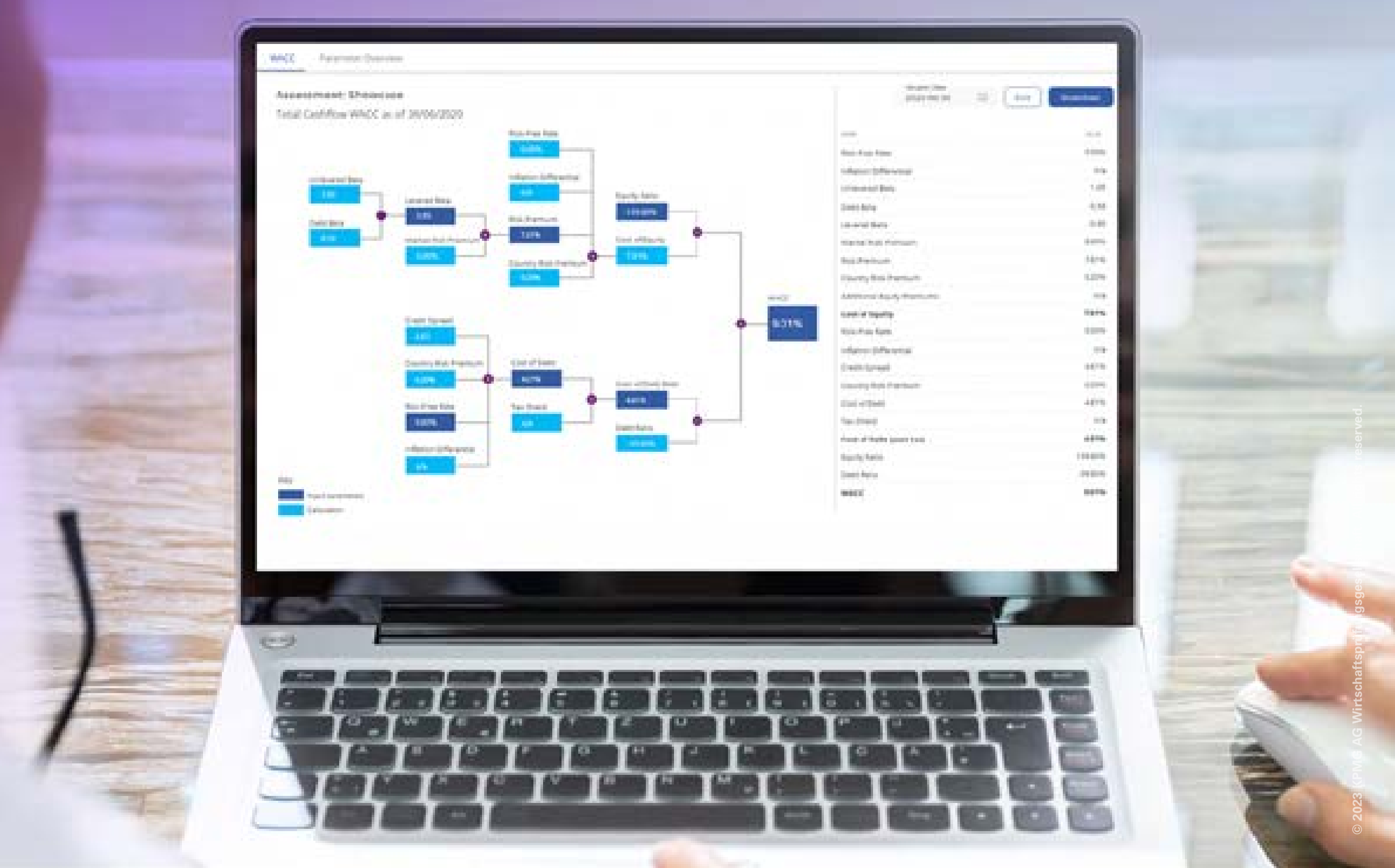

pany's uncertain planning premises are taken as a risk (e.g., assumptions about exchange rates, raw material prices or the sales growth rate). The risks are described by suitable probability distributions; for example, an exchange rate fluctuation with a normal distribution, or the uncertain growth rate specifying (a) minimum value, (b) most probable value and (c) maximum value (triangular distribution or beta-percent distribution).[35] All risks are assigned to the corporate planning items (integrated) in which they can trigger deviations from the plan. Risk aggregation (Monte Carlo simulation) based on this risk analysis is then used to calculate a sufficiently large representative number of future scenarios. In this way, the expected value and, consistently, the risk measure of cash flows or flow-to-equity can be derived.[36]

In the simple case study here, all the ways outlined, including risk analysis and risk aggregation, lead to similar results in the standard deviation of return (flow-to-equity) as the risk measure to be used to determine the cost of capital ($\mathbf{c}$).

The risk aggregation (risk simulation) allows quantification of the earnings risk, expressed by the standard deviation of earnings. This corresponds to the standard deviation of profit $\boldsymbol{\sigma}_{\text{Earning}}$ = € 3.1 million (the standard deviation of the past is € 3,47 million and thus similar, see Figure 2). The standard deviation of profit can be interpreted as a measure of planning certainty. This corresponds to the value at risk (e.g., at the 99% level), and in the case study is around € 13 million.

The correlation of the earnings (or changes in earnings) of companies to the market index is 0.5, which thus corresponds to the degree of risk diversification $\mathbf{d}$ (see Section III. 5. for derivation).

Equation (10) can be used to calculate the following risk-adjusted discount rate.[37] The equation converts the coefficient of variation ($\mathbf{V}$) derived from risk analysis and risk aggregation into an expected return corresponding to this risk, i.e., a risk-adequate discount rate ($\mathbf{c}$). The derivation is based on the method of "imperfect replication" briefly outlined above). In the case study, it is assumed that the correlation of earnings to the return (or earnings) of the market portfolio is as high as the correlation between the company's shares and the market portfolio, i.e., $\mathbf{d} = \boldsymbol{\rho} = \mathbf{0.5}$ (see Equation (3)). It is possible that some medium-sized entrepreneurs, who essentially own their company, will set $\mathbf{d} = \mathbf{1}$ when determining subjective decision values, i.e., neglect risk diversification effects.[38]

$$c = \frac{1 + r_f}{1 - \lambda \cdot V \cdot d} - 1 = \frac{1 + r_f}{1 - \lambda \cdot \frac{\sigma_{\text{Earning}}}{\text{Earning}_2^e} \cdot d} - 1 = \frac{1 + 3\%}{1 - 0.25 \cdot \frac{3.1}{9} \cdot 0.5} - 1 = 7.6\% \qquad (10)$$

with

$$\lambda = \frac{r_m^e - r_f}{\sigma_{r_m}} = \frac{8\% - 3\%}{20\%} = 0.25$$

and

$$V = \frac{\sigma_{\text{Earning}}}{\text{Earning}^e}$$

as coefficient of variation.

As can easily be seen, the ratio $\boldsymbol{\lambda}$ (Sharpe ratio) is exclusively derived from information that is also used in the CAPM (specifically beta factor, see Equations (2) and (3)). $\boldsymbol{\lambda}$ is a measure of the risk-return profile of alternative investment opportunities. Accordingly, a $\boldsymbol{\lambda}$ of 0.25 expresses that an additional return of 0.25% can be expected on the capital market per unit of increased risk. When evaluating the company, its risk-return profile is compared with that of the selected alternative investment options; in this case, government bonds and a broad stock market index (e.g. MCSI World).

In our case study, the risk-adjusted cost of capital deviates significantly from that determined using the CAPM. The "implied beta factor" $\boldsymbol{\beta}'$, which can be calculated for comparison purposes, is:

$$\beta' = (c - r_f) / (r_m^e - r_f) = 0.92$$

The reason for this is that the risk-adequate cost of capital precisely considers the risks (which are in themselves relevant to valuation) of a company's future earnings and cash flows, and not, as in the CAPM, the risks from (historical) stock price fluctuations, which are mainly significant for a shareholder investing in the short term.

For the value in the initial situation, considering the unbiased earnings of € 8.88 million, the probability of insolvency (1.3%) and the cost of capital (7.6%), the following result is obtained. $\mathbf{Value_2}$ stands for the value of variant 2, which corresponds to the risk-adequate approach (simulation-based valuation).

$$\text{Value}_2\left(\text{Earning}_2^e\right) = \frac{\text{earning}_2^e \cdot (1 - p)}{c + p} = \frac{9 \cdot (1 - 1.3\%)}{7.6\% + 1.3\%} = 99.8 \qquad (11)$$

35 Wehrspohn/Ernst, When Do I Take Which Distribution? A Statistical Basis for Entrepreneurial Applications, 1st ed. 2022.

36 See the relevant risk management methods in Gleißner, op. cit. (footnote No. 10); Ernst, op. cit. (footnote: 17): 1-13.

37 Derivation e.g. in Gleißner, op. cit. (footnote: 10): 1243-1258.

38 See Kerins/Smith/Smith, Opportunity Cost of Capital for Venture Capital Investors and Entrepreneurs, JFQA, no. 6 (2004): 385-405 and the total beta approach that assumes d = 1.

4. Strategy evaluation: risk-adequate evaluation of an option for action

So far, the status quo of the company has been considered. The calculated (risk-adequate) company value of € 99.8 million is to be interpreted as a "benchmark" (or hurdle rate) for the strategy assessment, i.e., the strategic action to now be evaluated. Such a course of action makes sense precisely if it increases the sustainable success of the company, i.e., leads to an improvement in the risk-return profile. The basis of the evaluation is now an alternative business plan, in which the effects of the planned measures on sales, costs and capital commitment are taken into account ("what-if analysis"). In the case study, it is assumed that the "internal" measure in the value chain will have no impact on sales. The capital commitment is also considered to be essentially unchanged.

From discussions and negotiations with the potential outsourcing partners, a significant improvement in profitability of one million euros is forecast.

A structured risk analysis, which is not presented in detail here, shows that outsourcing has advantages and disadvantages in terms of the risk position. The advantage is that production-related risks (e.g., due to machine failure) are eliminated within the company because the corresponding activities are no longer carried out. Another advantage is that previously largely fixed costs are substituted by variable costs based on sales. However, a quantitative risk analysis also shows the downsides of outsourcing: an on-site visit to the production facility of the potential outsourcing partner by a team of experts with specialists from production, quality assurance and logistics showed that, due to a largely lack of redundancy, the technical insolvency probabilities at the partner company are higher than those previously seen at the company itself. In addition, a rating analysis of this company's key financial figures reveals a non-negligible insolvency risk, which could lead to the loss of a key supplier that is virtually impossible to replace in the short term. From publicly available data on equity ratio and profitability, a "B" rating is estimated, which implies an insolvency probability of no less than 5% per year.

This and other information on the changes in opportunities and threats (risks) in the event of outsourcing are now taken into account in the above-mentioned alternative planning. Then, by means of Monte Carlo simulation and risk aggregation, the change in the realistic range of the company's earnings and cash flows is shown. This occurs if the measure being assessed is implemented. In the case study, the expected increase in earnings to € 10 million $\mathrm{earning}_3^e$ is initially confirmed because, in addition to risks, some opportunities (further cost-saving opportunities) are also identified in the risk analysis. In addition, however, the risk aggregation shows a significant increase in the standard deviation of earnings (the risk measure) from € 3.1 million to € 4.2 million. This increase is caused by possible additional costs and lost sales in the event of a technical production stoppage (interruption of operations), or even insolvency of the future key supplier. However, the effects will not be so extreme as to have a significant impact on the company's own insolvency probability and rating. This means that an insolvency probability of p = 1.3% is still assumed.

Based on the increased predicted profitability and the simultaneously increased level of risk, we can now calculate the risk-adjusted cost of capital ($\mathbf{c'}$) that would result in the case of an outsourcing decision (also $\mathbf{d}$, i.e., the share of risks to be borne is assumed to be constant, which can be examined as part of a detailed analysis):

$$c' = \frac{1+r_f}{1-\lambda\cdot\frac{\sigma_{\mathrm{Earning}_3}}{\mathrm{Earning}_3^e}\cdot d} - 1 = \frac{1+3\%}{1-0.25\cdot\frac{4.2}{10}\cdot 0.5} - 1 = 8.7\% \tag{12}$$

and

$$\mathrm{Value}_3\left(\mathrm{Earning}_3^e\right) = \frac{\mathrm{Earning}_3^e}{c'+p} = \frac{10\cdot\left(1-1.3\%\right)}{8.7\%+1.3\%} = 98.7 \tag{13}$$

$\mathbf{Value_3}$ stands for the value of variant 3, which corresponds to the risk-adequate approach taking into account the strategic option. Here, the effect of outsourcing on the company value as a measure of success can be seen directly. As we can see, the company value falls from € 99.8 million to € 98.7 million. The reason for this is that the risk-return profile deteriorates slightly. The increase in aggregate total risk and thus in the cost of capital more than compensates for the expected increase in profitability as a result of outsourcing. As mentioned above, this measure would not unduly affect the company's security of tenure (credit rating). However, it does not make economic sense when weighing up return and risk (cf. the following summary in Table 2).

5. The derivation of the diversification factor d

Up to now, it has been assumed for simplicity that the risk diversification factor $\mathbf{d}$ remains approximately unchanged. This assumption is uncontroversial if the valuation subject considers all risks to be relevant for valuation, i.e., sets $\mathbf{d}$ = 1. If, however, in line with the CAPM assumption system, only the risks that cannot in principle be diversified are considered in the valuation calculation, changes in the degree of risk diversification are possible.[39] They are to be expected if the ratio between systematic and unsystematic risks changes.

39 For risk diversification see Gleißner/Wolfrum, Cost of capital and valuation with imperfect diversification and unsystematic risks, working paper, 2009, SSRN-ID 1437629 (last access 05.05.2023).

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Table 2: Evaluation of a strategic option for action

	Status quo	Strategic option
Earning	€ 9 million	€ 10 million
Risk (coefficient of variation earning)	34%	42%
Cost of capital	7.6%	8.7%
Rating forecast (probability of default)	1.3%	1.3%
Rating (stress scenario)	BB	BB
Value (in € million)	99.8	98.7
Strategic fitting		Yes, but worse risk-return profile

Table 3: Profit development of the company and all companies in the market index (in billions of €)*

	t(-13)	t(-12)	t(-11)	t(-10)	t(-9)	t(-8)	t(-7)	t(-6)	t(-5)	t(-4)	t(-3)	t(-2)	t(-1)	t(0)
Profit Company	6.6	4	4	-1	6	4.5	5.5	6	5	-4	3	7	8	9
Profit Market index	47	12	-13	14	45	69	86	99	33	4	87	80	87	

*Source: Gleißner, op. cit. (footnote: 8).

In the simplest case, we can assume exactly one systematic risk factor of the exogenous environment for the calculation of d, such as, for example, the earnings fluctuations of all companies (e.g., of an economy), which are essentially caused by the business cycle (the CAPM also assumes exactly one risk factor, in contrast to the arbitrage pricing theory[40] (APT). There, the uncertain return of the market portfolio ($\mathbf{r}_m$) is usually used as a risk factor due to a "capital market-oriented" view.[41] It is possible to consider several exogenous risk factors – e.g. complementary inflation, exchange rate, commodity price – in an extended "risk factor model". In the case study, we now consider a "corporate earnings index" as the only systematic risk factor whose risk effects cannot be eliminated even for a diversified valuation subject (owner).

We can verify the estimate of risk measure and risk diversification degree $\mathbf{d}$ with historical profit fluctuations (see Figure 2). The standard deviation of the (trend-adjusted) past profit fluctuations of the company itself (see Table 3 and Figure 2) is 3.47, which is quite similar to the result of the risk aggregation. The correlation of the company's profits (or profit changes) to the profits of all companies in the market index (in € billions, source: Boerse Online database), which can also be derived from 2, is about 0.5 for the profit change rate (or 0.6 for the profits themselves), which roughly corresponds to the assumed risk diversification degree $\mathbf{d}$.

40 See also Fama/French, A five-factor asset pricing modell, Journal of Financial Economics, vol. 116, no. 1 (2015): 1-22.

41 See e.g. Rubinstein, op. cit. (footnote: 16): 61-69 and McConaughy/Covrig, Owners' Lack of Diversification and Cost of Equity Capital for Closely Held Firm, Business Valuation Review, vol. 26, no. 4 (2007): 115-120.

In a continuation of the case study, we now examine whether the strategic action option under consideration would lead to a change in the risk diversification factor $\mathbf{d}$.

From the quantitative risk analysis of the company, in particular the consideration of the uncertain assumptions in the planning model, it is determined that, essentially (and statistically significantly), sales only fluctuate as a function of overall economic demand (GDP), and that profit depends on all companies. The direct dependence of other planning items, such as material and personnel expenses, on this general exogenous risk factor is not statistically significant and is neglected. From the simulation-based risk aggregation, it thus follows that the valuation-relevant earnings considered here are also dependent on this risk factor. The various event-oriented risks in the value chain and the support processes (operational performance risks) are independent of the overall economic development expressed by the general risk factor. However, it is clear from empirical studies that the probability of insolvency of companies – in this case, of a key supplier – depends on the general earnings development of companies (as theoretically expected). This fact is captured in the risk aggregation model by linking the insolvency probability $\mathbf{p}^{\text{Supplier}}$ with the earnings index (or with GDP).

The measures explained above would change the company's risk profile. In order to be able to use this additional information to determine the change in the risk diversification factor from $\mathbf{d}$ to $\mathbf{d'}$, two risk aggregations are carried out in each of the two strategy assessment cases considered – (1) status quo and (2) status quo plus the

Table 4: Risk diversification factor d

	Overall risk		Risk diversification factor
	All risks including exogenous risk factor (a)	Only exogenous risk factor (b)	Proportion of systematic risks (d or d') (b/a)
Status quo	3.1	1.55	d = 1.55/3.1 = 0.50
Status quo plus measure	4.2	2.2	d' = 2.2/4.2 = 0.52

strategic action option – namely, once (a) with and once (b) without considering the dependency of sales, and thus of the company's earnings, on the general risk factor "company earnings" (or GDP). The results are shown in Table 4.

We can see the (slight) change in the risk diversification factor from $\mathbf{d}$ = 0.5 to $\mathbf{d}'$ = 0.52 by the measures (strategies) to be evaluated.

In the case study, therefore, there is a slight change in the degree of risk diversification $\mathbf{d}$ if a company is valued from the perspective of a (perfectly) diversified economic entity – as mentioned, this is of course irrelevant when considering all company risks ($\mathbf{d}$ = 1), as happens with the total beta model.[42] In the case under consideration, the change is also relatively small, and also tends to be "unfavorable", so that the above assessment of negligibility can at least be justified in principle. However, a review of the facts is of course useful if additional safeguarding is desired. In the case study, this relatively small effect results from the fact that, although the overall scope of risk has increased due to additional "unsystematic" risks, at the same time, a significant systematic risk – the probability and thus the expected impact of the key supplier defaulting – has increased.

The approach explained here is a simple introduction to the use of risk factor models, and also shows how changes in the degree of risk diversification (forward-looking) can be mapped. Here, too, it is important to present assumptions made (simplifications) in a transparent manner (in the example, only a cursory explanation of the change in the risk aggregation model was given). It should be emphasized that in this way, changes in risk diversification effects due to measures are addressed. In the traditional derivation of the CAPM beta factor for a capital market-oriented valuation, only historical information is used, i.e., it is implicitly assumed that the risk diversification factor remains unchanged. Changes in risk diversification effects (in this case, the correlation of the company's earnings or returns to the market portfolio return) are thus ignored in traditional capital market-oriented valuation methods in valuation practice, especially in strategy valuation.

42 Cf. Ernst/Gleißner, op. cit. (footnote: 9).

IV. Conclusion and implications for practice

In this paper, the method of simulation-based business valuation, based on the so-called semi-investment valuation theory, was explained and illustrated in possible applications by means of a case study. As shown, the presented method is an alternative to a DCF valuation based on the CAPM, which uses essential known building blocks derived from the method box of business valuation (like the DCF). The great advantage of the method is that the perfection of the capital market or the availability of capital market data on the valuation object (company) is not assumed, and rating and financing restrictions are also included in the valuation calculation (insolvency risks). A consistent derivation of the expected values of cash flows (or flow-to-equity) and the cost of capital to be used as a basis for the DCF method considers the opportunities and threats (risks) of a company. Based on the stringent identification and quantification of the risks, as well as their aggregation, by means of Monte Carlo simulation, it is possible to derive risk-adjusted discount rates. It is also possible to evaluate and compare different strategic courses of action or investment options of a company, whereby their different risk contents are taken into account in the evaluation calculation.

Glossary

Input-oriented valuation variant

The "input-oriented" valuation considers the rating and financing restrictions of the company. The risk is understood as a "possible loss" (e.g. measured by VaR). This valuation concept is called the "risk coverage approach" and can also be derived using the "incomplete replication" method.

Insolvency risk

The concept of insolvency risk is derived from risk theory and the conceptual understanding of risk in general. Accordingly, insolvency risk describes the possibility that insolvency may occur as a result of the uncertain future development of the company (with a probability > 0 over the entire future). To measure the level of insolvency risk, risk measures are needed, as for other risks.

The simplest insolvency risk measure is the insolvency probability. This is formally a lower partial moment of degree 0. Such LPM0 risk measures only indicate the probability of a certain event occurring, namely falling below a threshold.

Output-oriented valuation variant

The "output-oriented"/semi-investment-theoretical valuation methods (as well as the traditional methods of a finance-theoretical (capital market-oriented) valuation (e.g. with the CAPM)) are based on an understanding of risk, whose risk measures express the extent of possible deviations from the expected value of the payments (especially often the standard deviation). In the case of principally "tradable" (e.g. listed) investments, the standard deviation (or the DVaR or relative VaR) is used as a risk measure (measure of plan deviations) because any deviation from the expected value of cash flows or earnings triggers a reduction in the value of equity (rather than losses). This is referred to as an "output-oriented" valuation, in which only the uncertain outcome of the future – and not the initial situation/assets (in t=0) – is included in the valuation.

Rating

Rating is understood as credit rating, more precisely as issuer rating (which is to be distinguished from issue rating). A rating grade (AAA, A, BBB, BB, B) corresponds to a probability of insolvency (or default). This can be estimated simply based on financial ratios of the company (e.g. equity ratio and return on capital employed, ROCE). A more precise assessment is possible if the findings from risk analysis and risk aggregation are also taken into account (simulation-based rating forecasts). Accordingly, the rating provides information on the level of insolvency risk.

Risk-adequate valuation

Risk-adequate valuation allows the value of an uncertain cash flow to be determined based on the risk content of the cash flow expressed by a risk measure (such as standard deviation, coefficient of variation, or value-at-risk). A risk-adequate valuation of a company thus requires information from an analysis and aggregation of the company's risks (opportunities and threats); but not information about the risk of the company's shares (as expressed in the beta factor of the CAPM).

Risk coverage approach

The risk coverage approach is a special input-oriented valuation method that is used when capital market "imperfections" are particularly severe. The risk coverage approach is applied when the valuation object is hardly tradable (saleable) and the valuation subject is not well diversified. Risk is understood as a possible loss (equity requirement, thus formally a value at risk). The method illustrates the importance of risk analysis and the link between valuation and risk-based financing in an imperfect capital market with financing restrictions. Constraints on risk coverage potential, specifically equity, imply that potential losses, i.e., equity requirements, are used as a measure of risk.

Semi-investment theory valuation theory

Semi-investment theory valuation is based on the ideas of investment theory valuation and accepts simplifications that allow, for example, a company valuation based on the discounted cash flow calculation. Valuation equations are derived using the "incomplete replication" method, which does not require any capital market information about the valuation object and does not assume perfect capital market. In contrast to investment theory valuations, only two alternative investment options (a risk-free investment and an available stock market index) are considered for the valuation (similar to the CAPM). This simplifies the valuation (in particular, no optimization procedures are required). The derivation of the valuation equation (and discount rates based on it) is based on one assumption: two payments at the same point in time have the same value if they match in expected value and selected risk measure. Valuation equations are derived using "imperfect replication".

Simulation-based valuation

The central characteristics of a simulation-based valuation – (1) the consideration of business risks and (2) the use of Monte Carlo simulation – do not initially imply a commitment to a specific valuation theoretical framework. The use of the methods is possible in (1) an investment-theoretical valuation, (2) a semi-investment-theoretical valuation using "imperfect replication" and (3) also in a capital market-oriented valuation based on the CAPM.

Unbiased plan value

The basis of the company valuation in the DCF calculation are "unbiased" plan values, i.e. plan values that can be realized "on average" of the risk-related possible future scenarios. The calculation of plan values that are unbiased presupposes knowledge of existing opportunities and threats (risks) through a risk analysis. Unbiased plan values (expected values of cash flows) are lower than the ambitious target values usually set by companies for the purpose of corporate management. Unbiased plan values are also called simulation-based plan values. ◆

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Beta Estimation under Thin Trading Conditions

Estimates on betas may be distorted by thin (infrequent) trading effects, yielding incorrect estimates. Finance literature has proposed numerous techniques to eradicate the effects of thin trading, ranging from (il-)liquidity indicators indicating distortions in beta estimates to beta correction procedures directly correcting them in the traditional market model. This article provides an overview of comprehensive sets of 16 popular (il-)liquidity indicators and 10 popular beta correction procedures. Subsequently, these (il-) liquidity indicators and beta correction procedures are examined according to superiority in terms of accuracy (predictive ability) among themselves as well as against each other. The results indicate (i) the (il-)liquidity indicators to generally outperform the beta correction procedures in small as well as in large stock markets, across different levels of thin trading as well as across different levels of risk (beta magnitudes) and, (ii) the Illiquidity (Amihud-Hasbrouck) Indicator, the Return-to-Turnover Indicator as well as the Trade-to-Trade Method to dominate.

I. Introduction

Estimates on beta as well as on peer group betas valuing private firms may be distorted by thin (infrequent) trading effects, yielding incorrect estimates on the cost of capital along with incorrect discount rates and, thus, resulting in incorrect values. Therefore, examining distortions in beta estimates is a question of interest to investors, valuation analysts and the academic community. Finance literature indicates that OLS beta coefficients are strongly affected by thin trading. In general, three crucial statistical implications of thin trading were identified: (i) The distribution of thinly traded stock returns tends not to be normally distributed, illustrated by the leptokurtosis of highly peaked distributions, (ii) the irregular gaps in the time series due to non-trading reduce the accuracy of the beta estimates and, (iii) the unreliability of the beta estimates leads to distortions in measuring portfolio returns.[1]

Distortion of beta induced by thin trading has been examined according to the three dimensions bias (efficiency), accuracy (predictive ability) and stability (stationarity). The main body of research has well documented that OLS beta estimates suffer from significant bias problems introduced by frictions in the trading process that delay the adjustment of a stocks' price to informational change,[2] with the resulting underestimation of the covariance of the time series of stock returns and the market index causing the OLS beta estimate to be downwards biased. Two sources of this downwards bias – trading delays and price adjustment delays – have been well documented,[3] being related since prices cannot adjust to reflect a shift in the market equilibrium (price adjustment) without a transaction occurring (trading). Inducing downwards bias, thin trading itself can stem from three sources: (i) First, there may be days when the stock is not traded at all, and, therefore, there is no information available about its returns. (ii) Second, there may be days with a low trading volume, inducing stock's price return to be hardly representative of what would have occurred if the liquidity were adequate. (iii) Finally, there may be days with a reasonable total trading volume, but with a low volume at the end of the day distorting the closing prices employed in determining the returns. In this article, the various sources of thin trading are not separated, since it would not materially impact the results.

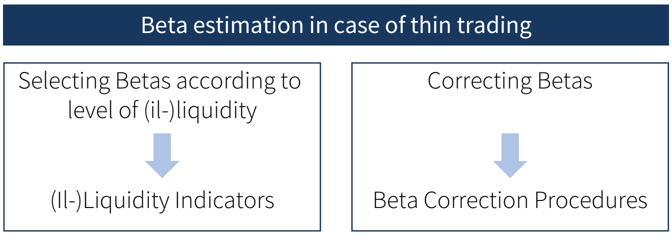
II. Techniques to Eradicate the Effects of Thin Trading

1. Overview

Numerous techniques have been proposed in finance literature to eradicate the effects of thin trading, ranging from (il-)liquidity indicators indicating potential distortions in beta estimates caused by low liquidity to beta correction procedures aiming at directly correcting them.[4]

While beta correction procedures are uniformly based on low-frequency (inter-day) pricing data, the (il-)liquidity indicators may be categorized according to frequency of data (low-frequency (inter-day) versus high-frequency (intra-day) data) as well as the type of data (price and volume data versus bid-ask spread data) employed. In this

Figure 1: Beta estimation in case of thin (infrequent) trading



Sources of thin trading:
- Complete days without trading
- Days with low trading volume
- Days with reasonable but low end-of-day trading volume

Figure 2: Data employed for (il-)liquidity indicators and beta correction procedures

Data Frequency Type of Data	Low-frequency (inter-day) data	High-frequency (intra-day) data
Price (and volume) data	Low-frequency price impact (il-)liquidity indicators Beta Correction Procedures	
Bid-ask spread data		High-frequency bid-ask spread (il-)liquidity indicators

1 Strebel, Thin Trading, Market Efficiency Tests and the Johannesburg Stock Exchange: A Rejoiner, The Investment Analysts Journal, vol. 7, no. 12 (1978): 29-30.

2 See e.g. Cloete/de Jonah/de Wet, Combining Vasicek and Robust Estimator for Forecasting Systematic Risk, Investment Analysts Journal, vol. 31, no. 55 (2001): 37-44.

3 See e.g. McInish/Wood, Adjusting for Beta Bias: An Assessment of Alternate Techniques: A Note, The Journal of Finance, vol. 41, no. 1 (1986): 277-286.

4 Furthermore, various restriction filters have been proposed to reduce the effect on an illiquid model, including (but not being limited to) price filters, trading filters, market capitalization filters and, stock exchange filters; see e.g. Basiewicz/Auret, Another Look at the Cross-Section of Average Returns on the JSE, Investment Analysts Journal, vol. 38, no. 69 (2009): 23-38; Strugnell/Gibert/Kruger, Beta, Size and Value Effect on the JSE, 1994-2007, The Investment Analysts Journal, vol. 40, no. 74 (2011): 1-17.

article, only (il-)liquidity indicators employing low-frequency (inter-day) data are considered.

2. (Il-)liquidity Indicators

Finance research has demonstrated that illiquidity appears to be one of the most important market frictions that impact asset prices,[5] demonstrating that significant price discounts exist for less liquid, otherwise comparable assets.[6] Liquidity is generally described as the ability to trade large quantities quickly at low cost and with little price impact. This definition highlights four dimensions to liquidity: (i) price impact, (ii) trading quantity, (iii) trading frequency (discontinuity, speed), and (iv) trading cost.[7] Since liquidity itself cannot be observed, numerous (il-)liquidity indicators have been proposed, each emphasizing differently on the dimensions of liquidity and being exposed to individual benefits and shortcomings.

The Illiquidity (Amihud) Indicator[8] captures the daily price response (absolute return) associated with one unit of monetary trading volume at that day, indicating the daily price impact of the order flow. Since the sample distribution of the Illiquidity (Amihud) Indicator usually has outliers, the Illiquidity (Amihud-Hasbrouck) Indicator[9] was proposed by taking its square root. Both indicators focus on the price impact dimension of liquidity. Furthermore, since the previous two indicators singularly emphasize on the price impact dimension of liquidity and, hence, neglect its trading-frequency dimension, the Adjusted Illiquidity (Amihud) Indicator was introduced.[10] It combines the virtues of the original Illiquidity (Amihud) Indicator and a non-trading frequency measure (correlating with the bid-ask spread and emphasizing on the trading frequency dimension of liquidity). All three indicators are proxies for illiquidity, since, by measuring the impact of a unit of monetary trading volume on stock's return, they imply that the higher the response of returns, the more illiquid (less liquid) the stock. Therefore, they are negatively related with liquidity and positively related with the risk premium of liquidity.

The formula for the Illiquidity (Amihud) Indicator is denoted by:

$$\mathrm{ILLIQI}_i^A = \frac{1}{T_i}\sum_{t=1}^{T_i}\frac{\left|r_{i,t}\right|}{\mathrm{MTV}_{i,t}} \tag{1}$$

ILLIQI_i^A Illiquidity (Amihud) Indicator of stock i
$\left|r_{i,t}\right|$ absolute daily return of stock i at day t
$\mathrm{MTV}_{i,t}$ monetary trading volume of stock i at day t
T_i number of positive volume trading days of stock i in the total measurement period

The formula for the Illiquidity (Amihud-Hasbrouck) Indicator is denoted by:

$$\mathrm{ILLIQI}_i^{AH} = \frac{1}{T_i}\sqrt{\sum_{t=1}^{T_i}\frac{\left|r_{i,t}\right|}{\mathrm{MTV}_{i,t}}} \tag{2}$$

ILLIQI_i^{AH} Illiquidity (Amihud – Hasbrouck) Indicator of stock i
$\left|r_{i,t}\right|$ absolute daily return of stock i at day t
$\mathrm{MTV}_{i,t}$ monetary trading volume of stock i at day t
T_i number of positive volume trading days of stock i in the total measurement period

The formula for the Adjusted Illiquidity (Amihud) Indicator is denoted by:

$$\mathrm{AILLIQI}_i^A = \left[\ln\left(\frac{1}{T_i}\sum_{t=1}^{T_i}\frac{\left|r_{i,t}\right|}{\mathrm{MTV}_{i,t}}\right)\right]\left(1+\mathrm{ZV}_i\right) \tag{3}$$

$\mathrm{AILLIQI}_i^A$ Adjusted Illiquidity (Amihud) Indicator of stock i
$\left|r_{i,t}\right|$ absolute daily return of stock i at day t
$\mathrm{MTV}_{i,t}$ monetary trading volume of stock i at day t
T_i number of positive volume trading days of stock i in the total measurement period
ZV_i percentage of zero-volume trading days of stock i in the total measurement period

5 See e.g., elaborating upon the relationship between liquidity and expected returns, Amihud/Mendelson, Asset pricing and the bid-ask spread, JFE, vol. 17, no. 2 (1986): 223-249; Brennan/Subrahmanyam, Market microstructure and asset pricing: On the compensation for illiquidity in stock returns, JFE, vol. 41, no. 3 (1996): 441-464; Brennan/Chordia/Subrahmanyam, Alternative factor specifications, security characteristics, and the cross-section of expected stock returns, JFE, vol. 49, no. 3 (1998): 345-373; Jacoby/Fowler/Gottesman, The capital asset pricing model and the liquidity effect: A theoretical approach, Journal of Financial Markets, vol. 3, no. 1 (2000): 69-81; Jones, A Century of Stock Market Liquidity and Trading Costs. Working Paper (2002): 1-46; Acharya/Pedersen, Asset pricing with liquidity risk, JFE, vol. 77, no. 2 (2005): 375-410.

6 See e.g. Heaton/Lucas, Evaluating the Effects of Incomplete Markets on Risk sharing and Asset Pricing, JPE, vol. 104, no. 3 (1996): 443-487; Vyanos, Transaction Costs and Asset Prices: A Dynamic Equilibrium Model, Review of Financial Studies, vol. 11, no. 1 (1998): 1-58; Lo/Mamaysky/Wang, Asset Prices and Trading Volume under Fixed Transaction Costs, JPE, vol. 112, no. 5 (2004): 1054-1090.

7 See e.g. Zhang/Yang/Su/Zhang, Liquidity premium and the Corwin-Schultz bis-ask spread estimate, China Finance Review International, vol. 4, no. 2 (2014): 168-186.

8 Amihud, Illiquidity and stock returns: cross-section and time-series effects, Journal of Financial Markets, vol. 5, no. 1 (2002): 31-56. It is utilized as an indicator of stock market liquidity in various settings e.g. by Amihud/Mendelson/Lauterbach, Market microstructure and securities values: Evidence from the Tel Aviv Stock Exchange, JFE, vol. 45, no. 3 (1997): 365-390; Acharya/Pedersen, op. cit. (footnote No. 5): 375-410; Amihud/Hameed/Kang/Zhang, The illiquidity premium: International evidence, JFE, vol. 117, no. 2 (2015): 350-368; Chordia/Huh/Subrahmanyam, Theory-Based Illiquidity and Asset Pricing, The Review of Financial Studies, vol. 22, no. 9 (2009): 3629-3668.

9 Hasbrouck, Trading Costs and Returns for US Equities: The Evidence from Daily Data, Working Paper (2005): 1-42; Hasbrouck, Trading costs and returns for US equities: Estimating effective costs from daily data, The Journal of Finance, vol. 64, no. 3 (2009): 1445-1477.

10 Kang/Zhang, Measuring liquidity in emerging markets, Pacific-Basin Finance Journal, vol. 27 (2014): 49-71.

The Liquidity (Amivest) Indicator[11] indicates the ratio of the average monetary trading volume to non-zero absolute returns (since the ratio is only defined for non-zero returns), capturing the monetary trading volume associated with a unit change in stock price. As with the Illiquidity (Amihud) Indicator, since the sample distribution of the Liquidity (Amivest) Indicator usually has outliers, the Liquidity (Amivest-Hasbrouck) Indicator was proposed by again taking its square root.[12] Furthermore, the log form of the Liquidity (Amivest) Indicator was introduced.[13] All three indicators are proxies for liquidity. Therefore, they are positively related with liquidity and negatively related with the risk premium of liquidity. They essentially are reciprocal to the Illiquidity (Amihud) Indicator and the Illiquidity (Amihud-Hasbrouck) Indicator and, hence, singularly emphasize on the price impact dimension of liquidity. However, since the latter exclude zero-volume trading days, whereas the former exclude zero-return trading days, and there normally occur more observations of zero-return days than zero-volume days, they may yield different results.

The formula for the Liquidity (Amivest) Indicator is denoted by:

$$\mathrm{LIQI}_i^{A} = \frac{1}{T_i} \sum_{t=1}^{T_i} \frac{\mathrm{MTV}_{i,t}}{\left|r_{i,t}\right|} \tag{4}$$

LIQI_i^{A} Liquidity (Amihud) Indicator of stock i
$\left|r_{i,t}\right|$ absolute daily return of stock i at day t
$\mathrm{MTV}_{i,t}$ monetary trading volume of stock i at day t
T_i number of positive volume trading days of stock i in the total measurement period

The formula for the Liquidity (Amivest-Hasbrouck) Indicator is denoted by:

$$\mathrm{LIQI}_i^{AH} = \frac{1}{T_i} \sqrt{\sum_{t=1}^{T_i} \frac{\mathrm{MTV}_{i,t}}{\left|r_{i,t}\right|}} \tag{5}$$

LIQI_i^{AH} Liquidity (Amihud – Hasbrouck) Indicator of stock i
$\left|r_{i,t}\right|$ absolute daily return of stock i at day t
$\mathrm{MTV}_{i,t}$ monetary trading volume of stock i at day t
T_i number of positive volume trading days of stock i in the total measurement period

The formula for the log form of the Liquidity (Amivest) Indicator is denoted by:

$$\mathrm{LNLIQI}_i^{A} = \ln\left(\frac{1}{T_i} \sum_{t=1}^{T_i} \frac{\mathrm{MTV}_{i,t}}{\left|r_{i,t}\right|} \right) \tag{6}$$

LNLIQI_i^{A} log form of the Liquidity (Amihud) Indicator of stock i
$\left|r_{i,t}\right|$ absolute daily return of stock i at day t
$\mathrm{MTV}_{i,t}$ monetary trading volume of stock i at day t
T_i number of positive volume trading days of stock i in the total measurement period

Since all previous indicators carry a size bias, the Return-to-Turnover Indicator was proposed,[14] eliminating the size bias by replacing monetary trading volume with the turnover ratio. As a proxy for illiquidity, it emphasizes on multiple dimensions of liquidity, namely price impact, trading quantity and, trading frequency. It is a proxy for illiquidity. Therefore, it is negatively related with liquidity and positively related with the risk premium of liquidity. The formula is denoted by:

$$\mathrm{RTTI}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} \frac{\left|r_{i,t}\right|}{\mathrm{TR}_{i,t}} \quad \text{with} \quad \mathrm{TR}_{i,t} = \frac{\mathrm{QTV}_{i,t}}{\mathrm{SO}_{i,t}} \tag{7}$$

RTTI_i Return-to-Turnover Indicator of stock i
$\left|r_{i,t}\right|$ absolute daily return of stock i at day t
$\mathrm{TR}_{i,t}$ turnover ratio of stock i at day t (i. e., the ratio of quantity trading volume of stock i at day t ($\mathrm{QTV}_{i,t}$) to the number of shares outstanding of stock i at day t ($\mathrm{SO}_{i,t}$))
T_i number of positive volume trading days of stock i in the total measurement period

Note: Since the ratio is undefined for zero-volume days, the average is computed over all days with a non-zero turnover ratio, i. e., a non-zero quantity trading volume.

The Turnover Indicator[15] incorporates the holding period of the stock, and thus, trading activity, emphasizing on the

11 Amihud, op. cit. (footnote No. 8): 31-56. It is utilized in various settings e.g. by Cooper/Growth/Avera, Liquidity, exchange listing, and common stock performance, Journal of Economics and Business, vol. 37, no. 1 (1985): 19-33; Khan/Baker, Unlisted trading privileges, liquidity and stock returns, The Journal of Financial Research, vol. 16, no. 3 (1993): 221-236; Amihud/Mendelson/Lauterbach, op. cit. (footnote No. 8): 365-390; Berkman/Eleswarapu, Short-term traders and liquidity: a test using Bombay Stock Exchange data, JFE, vol. 47, no. 3 (1998): 339-355.

12 Hasbrouck, op. cit. (footnote No. 9): 1-42; Hasbrouck, op. cit. (footnote No. 9): 1445-1477.

13 Cooper/Growth/Avera, op. cit. (footnote No. 11): 19-33; Amihud/Mendelson/Lauterbach, op. cit. (footnote No. 8): 365-390.

14 Florackis/Gregorious/Kostakis, Trading frequency and asset pricing on the London Stock Exchange: Evidence from a new price impact ratio, JBF, vol. 35, no. 12 (2011): 3335-3350.

15 Amihud/Mendelson, op. cit. (footnote No. 5): 223-249; Haughan/Baker, Commonality in the determinants of expected stock returns, JFE, vol. 41, no. 3 (1996): 401-439; Chalmers/Kadlec, An empirical examination of the amortized spread. JFE, vol. 48, no. 2 (1998): 159-188; Datar/Naik/Radcliffe, Liquidity and Stock Returns: An Alternative Test, Journal of Financial Markets, vol. 1, no. 2 (1998): 203-219.

trading quantity and the trading frequency dimensions of liquidity. Representing a proxy for liquidity, various studies proved it to be positively related with stock liquidity and negatively related with the risk premium of liquidity.[16] The formula is denoted by:

$$TOI_i = \frac{1}{T_i}\sum_{t=1}^{T_i}\frac{MTV_{i,t}}{TMC_{i,t}} \tag{8}$$

TOI_i Turnover Indicator of stock i
$MTV_{i,t}$ monetary trading volume of stock i at day t
$TMC_{i,t}$ tradeable market capitalization of stock i at day t
T_i number of trading days of stock i in the total measurement period

The simple Monetary Trading Volume and the Quantity Trading Volume can be employed as liquidity indicators since, theoretically, the trading volume (or trading frequency) of a stock is an increasing function of its liquidity. Both indicators are measures of trading activity, emphasizing on the trading quantity dimension of liquidity. Therefore, they are positively related with liquidity and negatively related with the risk premium of liquidity. This positive relationship was confirmed by concluding that liquidity is directly related to how quickly investors can hedge their position,[17] documenting a strong cross-sectoral relationship between trading volume and various measures of bid-ask spread and market depth,[18] employing the log form of the indicator[19] and, evaluating its performance as compared to structural formulae.[20]

The formula for the Monetary Trading Volume Indicator is denoted by:

$$MTVI_i = \sum_{t=1}^{T_i} MTV_{i,t} \tag{9}$$

$MTVI_i$ Monetary Trading Volume Indicator of stock i
$MTV_{i,t}$ Monetary Trading Volume of stock i at day t
T_i number of trading days of stock i in the total measurement period

The formula for the Quantity Trading Volume Indicator is denoted by:

$$QTVI_i = \sum_{t=1}^{T_i} QTV_{i,t} \tag{10}$$

$QTVI_i$ Quantity Trading Volume Indicator of stock i
$QTV_{i,t}$ Quantity trading volume of stock i at day t
T_i number of trading days of stock i in the total measurement period

The Gamma Indicator[21] stresses the relationship between return and lagged order flow. It captures the reverse of the price impact of the previous day's order flow shock, assuming that order flow induces a price adjustment that initially overshoots and then reverses to real value. Therefore, it emphasizes on both, the price impact and the trading quantity dimension and, to some extent, the trading frequency (discontinuity) dimension of liquidity. The Gamma Indicator is a proxy for liquidity. Therefore, it is positively related with liquidity and negatively related with the risk premium of liquidity. The regression equation is denoted by:

$$r^e_{i,t} = \alpha_i + \beta_i r_{i,t} + \gamma_i \operatorname{sign}\left(r^e_{i,t-1}\right) MTV_{i,t-1} + \varepsilon_i$$
with
$$r^e_{i,t} = r_{i,t} - r_{m(i),t}$$
$$r^e_{i,t-1} = r_{i,t-1} - r_{m(i),t} \tag{11}$$

γ_i Gamma Indicator of stock i
$r^e_{i,t}$ excess return of stock i at day t
$r^e_{i,t-1}$ one day lagged excess return of stock i
$r_{i,t}$ daily return of stock i at dayt
$r_{i,t-1}$ one day lagged return of stock i
$r_{m(i),t}$ market respective capitalization weighted market return on stock i at day t
α_i intercept of the regression on stock i
β_i regression coefficient on the daily stock return
$\operatorname{sign}(\lambda)$ sign function taking the value 1 (-1) when l is positive (negative) and equals zero when l is zero
$MTV_{i,t-1}$ monetary trading volume of stock i one day lagged
ε_i regression error term

16 See e.g. Su/Mai, Liquidity and Asset Pricing: An Empirical Exploration of Turnover and Expected Returns on Chinese Stock Markets, Economic Research Journal, vol. 39, no. 2 (2004): 95-105; Chordia/Subrahmanyam/Anshuman, Trading activity and expected stock returns, JFE, vol. 59, no. 1 (2001): 3-32; Nguyen/Mishra/Prakash/Ghosh, Liquidity Asset Pricing under the Three-Moment CAPM Paradigm, The Journal of Financial Research, vol. 30, no. 3 (2007): 379-398.

17 See e.g. Stoll, The supply dealer services in security markets, The Journal of Finance, vol. 33 (1978): 1133-1151.

18 Brennan/Subrahmanyam, Investment analysis and price formation in securities markets, JFE, vol. 38, no. 3 (1995): 361-381; Brennan/Chordia/Subrahmanyam, op. cit. (footnote No. 5): 345-373; Datar/Naik/Radcliffe, op. cit. (footnote No. 15): 203-219; Chordia/Roll/Subrahmanyam, Commonality in liquidity, JFE, vol. 56, no. 1 (2000): 3-28.

19 Chordia/Subrahmanyam/Anshuman, op. cit. (footnote No. 16): 3-32.

20 Chordia/Huh/Subrahmanyam, op. cit. (footnote No. 8): 3629-3668.

21 Campbell/Grossman/Wang, Trading Volume and Serial Correlation in Stock Returns, QJE, vol. 108, no. 4 (1993): 905-939; Pàstor/Stambaugh, Liquidity Risk and Expected Stock Returns, JPE, vol. 111, no. 3 (2003): 642-685.

The LM Indicator (Liquidity Measure)[22] introduces the effect of the absence of trading on liquidity, thus, emphasizing on multiple dimensions of liquidity, namely trading quantity, trading frequency (trading speed and discontinuity) and trading cost, with particular emphasis on trading speed. Since it is a proxy for liquidity, it is positively related with liquidity and negatively related with the risk premium of liquidity. The formula is denoted by:

$$LMI_i = \left(ZV_i + \frac{\frac{1}{TOI_i}}{Deflator} \right) \frac{\overline{T_{m,i}}}{TN_i}$$

with

$$TOI_i = \frac{1}{T_i} \sum_{t=1}^{T_i} \frac{MTV_{i,t}}{TMC_{i,t}} \tag{12}$$

LMI_i LM Indicator of stock i
ZV_i number of zero-volume trading days of stock i in the total measurement period
TOI_i Turnover Indicator of stock i (see equation (8) for the Turnover Indicator)
$\overline{T_{m,i}}$ average of all stock trading days in the total measurement period in all stock markets m
TN_i number of days without trading of stock i in the total measurement period

Note: Deflator is chosen such that $0<((1/TOI_i)/Deflator)<1$ for all sample stocks uniformly. Since the number of trading days varies over time and for different stocks, multiplying by the factor $\overline{T_{m,i}}/TN_i$ standardizes the number of trading days to the average, making LMI_i comparable over time and across all stocks and, thus, serving as a tiebreaker for the situation where two stocks have the same number of zero daily volumes, but the one with the larger turnover rate should be more liquid.

The Zero Returns Indicator[23] emphasizes on the trading cost and trading frequency (trading discontinuity) dimension of liquidity. The underlying assumption is that informed traders will trade only when the gain from their private information is large enough to offset transaction cost, that is, if the stock liquidity is low, the high transaction cost will deter the trading from informed investors and therefore prevent private information from being revealed. The Zero Volume Indicator regards the number of days without trading (zero volume days), emphasizing on the trading quantity and the trading frequency (trading discontinuity) dimension of liquidity. Both indicators are proxies for illiquidity. Therefore, they are negatively related with liquidity and positively related with the risk premium of liquidity.

The formula for the Zero Returns Indicator is denoted by:

$$ZRI_i = \frac{\sum_t^T ZR_{i,t}}{T_i} \tag{13}$$

ZRI_i Zero Returns Indicator of stock i
$ZR_{i,t}$ trading days t with zero return of stock i
T_i number of trading days of stock i in the total measurement period

The formula for the Zero Volume Indicator is denoted by:

$$ZVI_i = \frac{\sum_t^T ZV_{i,t}}{T_i} \tag{14}$$

ZVI_i Zero Volume Indicator of stock i
$ZV_{i,t}$ days t without trading of stock i
T_i number of trading days of stock i in the total measurement period

Finally, being closely related to the Zero Returns Indicator and the Zero Volume Indicator, the Extended Zero Returns Indicator and the Extended Zero Volume Indicator replace the number of trading days, thus, similarly emphasizing on the trading quantity and the trading frequency (trading discontinuity) dimensions and, the trading cost dimension of liquidity, respectively. They can work as both, proxies for liquidity and illiquidity, depending on the proportion of the relative change of zero returns/volume days and the relative change of monetary trading volume.

The formula for the Extended Zero Returns Indicator is denoted by:

$$EZRI_i = \frac{\sum_t^T ZR_{i,t}}{\frac{\sum_t^T MTV_{i,t}}{T_i}} \tag{15}$$

22 Liu, A liquidity-augmented capital asset pricing model, JFE, vol. 82, no. 3 (2006): 6316-6371.

23 Lesmond/Ogden/Trzcinka, A new estimate of transaction costs, The Review of Financial Studies, vol. 12, no. 5 (1999): 1113-1141. It was employed in various settings e.g. by Lee, The world price of liquidity risk, JFE, vol. 99, no. 1 (2011): 136-161; Bekaert/Harvey/Lundblad, Liquidity and Expected Returns: Lessons from Emerging Markets, Review of Financial Studies, vol. 20, no. 6 (2007): 1783-1831; Lesmond/Schill/Zhou, The illusory nature of momentum profits. JFE, vol. 71, no. 2 (2004): 349-380.

Table 1: Dimensions captured and indication of (il-)liquidity indicators

(Il-)lquidity indicator	Dimensions of il-/liquidity					Indication	
	Price impact	Trading frequency		Trading Quantity	Trading cost	Liquidity	Illiquidity
		Speed	Discontinuity				
Illiquidity (Amihud)	✓						✓
Illiquidity (Amihud-Hasbrouck)	✓						✓
Adjusted Illiquidity (Amihud)	✓	✓					✓
Liquidity (Amivest)	✓					✓	
Liquidity (Amivest-Hasbrouck)	✓					✓	
Log Liquidity (Amivest)	✓					✓	
Return-to-Turnover	✓	✓	✓	✓			✓
Turnover		✓	✓	✓		✓	
Monetary Trading Volume				✓		✓	
Quantity Trading Volume				✓		✓	
Gamma	✓		✓	✓		✓	
Liquidity Measure		✓	✓	✓	✓	✓	
Zero Returns			✓		✓		✓
Zero Volume			✓	✓			✓
Extended Zero Returns			✓		✓	✓	✓
Extended Zero Volume			✓	✓		✓	✓

$EZRI_i$ Extended Zero Returns Indicator of stock i
$ZR_{i,t}$ trading days t with zero return of stock i
$MTV_{i,t}$ monetary trading volume of stock i at day t
T_i number of trading days of stock i in the total measurement period

The formula for the Extended Zero Volume Indicator is denoted by:

$$EZVI_i = \frac{\sum_t^T ZV_{i,t}}{\frac{\sum_t^T MTV_{i,t}}{T_i}} \tag{16}$$

$EZVI_i$ Extended Zero Volume Indicator of stock i
$ZV_{i,t}$ days t without trading of stock i
$MTV_{i,t}$ monetary trading volume of stockiat dayt
T_i number of trading days of stock i in the total measurement period

Table 1 summarizes the dimensions of ill-/liquidity captured as well as the indication addressed by the various (il-)liquidity indicators.

3. Beta Correction Procedures

Finance research has proposed numerous procedures correcting distortions in OLS beta estimates caused by thin trading. However, no consensus exists, whether the beta correction procedures generally yield improvements over OLS beta estimates in the traditional market model. Some studies on the Finnish and the Canadian market find the Trade-to-Trade Method and the Aggregated Coefficients Model to outperform (emphasizing accuracy) traditional OLS beta estimates.[24] Meanwhile, it was concluded that beta estimates corrected by the Aggregated Coefficients Model and the Triple Single Factor Model are less biased, but also less accurate, than OLS beta estimates.[25] Separating bias and standard error of the beta estimator, it was concluded that OLS betas do worst in terms of bias, but best in terms of standard error.[26] In contrary, on the U. S., the Swedish, the New Zealand, the Belgian and, the Australian market, no significant improvement was found, or the corrected beta

24 Luoma/Martikainen/Perttunen/Pynnonen, Different Beta Estimation Techniques in Infrequently Traded and Inefficient Stock Markets, Omega International Journal of Management Science, vol. 22, no. 5 (1994): 471-476; Brooks/Faff/Fry/Bissoondoyal-Bheenick, Alternative beta risk estimators in cases of extreme thin trading: Canadian Evidence, Applied Financial Economics, vol. 15, no. 18 (2005): 1251-1258.

25 Fowler/Rorke/Jog, A Bias-Correcting Procedure for Beta Estimation in the Presence of Thin Trading, The Journal of Financial Research, vol. 12, no. 1 (1989): 23-32.

26 Sercu/Vandebroek/Vinaimont, Thin-Trading Effects in Beta: Bias v. Estimation Error, JBFA, vol. 35, no. 9/10 (2008): 1196-1219.

estimates were even outperformed by the traditional OLS beta estimates.[27]

Being very simple in nature, the Repetition of Last Quote Procedure replaces prices of days without trading by the price of the last day of trading (this corresponds to the Lumped Returns Procedure assigning all multiperiod returns to the day the stock trades actually and setting all returns of the intervening non-trading days equal to zero).[28] The Uniform Quotes Procedure replaces the price of all days without trading by the arithmetic mean of the price of the last and the next day of trading. The Uniform Returns Procedure (Average Returns Procedure) replaces the returns of all days without trading by the geometric mean of the return of the last and the next day of trading, thus, allocating the realized return equally over all days in the interval.

The formula for the Repetition of Last Quote Procedure (Lumped Returns Procedure) is denoted by:

$$p_{i,t} = p_{i,t-d+j} \tag{17}$$

$p_{i,t}$ price of stock i at day t
d length (number of days) of the non-trading interval
j number of remaining days without trading at day t in the non-trading interval

The formula for the Uniform Quotes Procedure is denoted by:

$$p_{i,t} = \frac{p_{i,t-1} + p_{i,t+1+j}}{2} \tag{18}$$

$p_{i,t}$ price of stock i at day t
j number of remaining days without trading at day t in the non-trading interval

The formula for the Uniform Returns Procedure (Average Returns Procedure) is denoted by:

$$r_{i,t} = \sqrt[d+1]{\frac{p_{i,t+1+j}}{p_{i,t-d+j}}} \tag{19}$$

$r_{i,t}$ return of stock i at day t
d length (number of days) of the non-trading interval
j number of remaining days without trading at day t in the non-trading interval

The Triple Single Factor Model[29] employs three estimates of single factor models by running the market model with three separate regressions of stock returns on a lagging, a synchronous and a leading market return. It is built on the assumption that the infrequent trading bias is caused by the non-synchronicity problem arising in analyzing time series, since it is implicitly assumed that all stocks are traded consecutively and, therefore, the price reflects the latest information. Instead, returns are computed only in case a transaction occurred in consecutive time periods.

The closed-end formula for the beta estimator and the regression equations are denoted by:

$$\beta_i^{TSFM} = \frac{\beta_i^{-1} + \beta_i^{0} + \beta_i^{+1}}{1 + 2\rho_{r_{m(i)}}}$$

$$\begin{aligned} r_{i,t} &= \alpha_i^{-1} + \beta_i^{-1} \cdot r_{m(i),t-1} + \varepsilon_i \\ r_{i,t} &= \alpha_i^{0} + \beta_i^{0} \cdot r_{m(i),t} + \varepsilon_i \\ r_{i,t} &= \alpha_i^{+1} + \beta_i^{+1} \cdot r_{m(i),t+1} + \varepsilon_i \end{aligned} \tag{20}$$

β_i^{TSFM} corrected beta estimator on stock i employing the Triple Single Factor Model
β_i regression coefficient of the regression on stock i with a one day lagging (β_i^{-1}), a synchronous (β_i^{0}) and, a one day leading (β_i^{+1}) market return
$\rho_{r_{m(i)}}$ first order autocorrelation coefficient of the market return of the respective market index m on stock i
$r_{i,t}$ return of stock i at day t
α_i intercept of the regression on stock i with a one day lagging (α_i^{-1}) a synchronous (α_i^{0}) and, a one day leading(α_i^{+1}) market return
$r_{m(i)}$ return of the respective market index of stock i one day lagging ($r_{m(i),t-1}$), at day t ($r_{m(i),t}$) and, one day leading ($r_{m(i),t+1}$)
ε_i regression error term

The Generalized Single Factor Model generalizes the Triple Single Factor Model to longer non-trading intervals, suggesting using two leading and lagging terms.[30] The closed-end formula for the beta estimator (weighted sum of the estimated beta coefficients) and the regression equation are denoted by:

27 McInish/Wood, op. cit. (footnote No. 3): 277-286; Berglund/Liljeblom/Löflund, Estimating Beta on Daily Data for a Small Stock Market, JBF, vol. 13, no. 1 (1989): 41-64; Bartholdy/Riding, Thin Trading and the Estimation of Betas: The Efficacy of Alternative Techniques, The Journal of Financial Research, vol. 42, no. 2 (1994): 241-254; Beer, Estimation of risk on the Brussels Stock Exchange: Methodological issues and empirical results, Global Finance Journal, vol. 8, no. 1 (1997): 83-94; Davidson/Josev, The impact of thin trading adjustments on Australian beta estimates, ARJ, vol. 18, no. 2 (2005): 111-117.

28 The Repetition of Last Quote Procedure is employed e.g. by Serra/Martelanc, Estimation of betas of stocks with low liquidity, Brazilian Business Review, vol. 10, no. 1 (2013): 49-78.

29 Scholes/Williams, Estimating betas from nonsynchronous data, JFE, vol. 5, no. 3 (1977): 309-327; Bradfield, Investment Basics XL.VI. On Estimating the Beta Coefficient, Investment Analysts Journal, vol. 32, no. 57 (2003): 47-53.

30 Fowler/Rorke, Risk measurement when shares are subject to infrequent trading, JFE, vol. 12, no. 2 (1983): 279-283.

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$$\beta_i^{GSFM} = \sum_{d=-n}^{n} \omega_{i,d}\beta_{i,d}\ , n \in \{1,2\}$$

$$r_{i,t} = \alpha_{i,d} + \beta_{i,d} \cdot r_{m(i),t+d} + \varepsilon_i$$

with

$$\omega_{d=\pm1} = \frac{1+\rho_{r_{m(i),d=1}}}{1+2\rho_{r_{m(i),d=1}}} ; \omega_{d=\pm2} = \frac{1+\rho_{r_{m(i),d=1}} + \rho_{r_{m(i),d=2}}}{1+2\rho_{r_{m(i),d=1}} + 2\rho_{r_{m(i),d=2}}} \quad (21)$$

β_i^{GSFM} corrected beta estimator on stock i employing the Generalized Single Factor Model
d index on lagging and leading terms
$\beta_{i,d}$ regression coefficient of the regression on stock i with d days lagging and leading market returns
$r_{i,t}$ return of stock i at day t
$\alpha_{i,d}$ intercept of the regression on stock i with d days lagging and leading market returns
$r_{m(i),t+d}$ return of the respective market index of stock i with d days lagging and leading
ε_i regression error term
$\rho_{r_{m(i),d=1,2}}$ first and second order autocorrelation coefficients of the market return of the respective market index m on stock i, respectively

The Aggregated Coefficients Model[31] estimates beta by running a multiple regression with lagging, a synchronous and leading market returns as additional regressors simultaneously. The model requests determining the number of lagging and leading terms, being a considerable debate in literature. For example, Bartholdy/Riding[32] proposed one, two and three leading and lagging terms. McInish/Wood[33] proposed one lagging and leading, two lagging and leading as well as five lagging and up to 21 leading terms. Berglund/Liljeblom/Löflund[34] proposed one, two, five, and ten lagging and leading terms. Theoretically, the thinner a stock is traded, the larger the number of lagging and leading terms necessary to reduce the thin trading bias. But the reduction of the thin trading bias is achieved at the expense of its overall performance. For example, it has been shown that an increasing number of lagging and leading terms decreases the overall performance of the beta estimator, concluding that employing numerous lagging and leading terms is not justifiable.[35]

Therefore, Fowler/Rorke[36] recommend using two lagging and two leading terms. Furthermore, Dimson[37] concludes that, if the market is very frequently traded relative to a stock, the lagging terms are of much greater importance than the leading terms. In such cases, he advises using at least four lagging and only one leading term.

The closed-end formula for the beta estimator and the regression equation are denoted by:

$$\beta_i^{ACM} = \sum_{d=-n}^{n} \beta_{i,d}$$
$$r_{i,t,d} = \alpha_i + \sum_{d=-n}^{n} \beta_{i,d} r_{m(i),t+d} + \varepsilon_i \quad (22)$$

β_i^{ACM} corrected beta estimator on stock i employing the Aggregated Coefficients Model
d index on lagging and leading terms
$\beta_{i,d}$ regression coefficient of the regression on stock i with lagging, synchronous and leading interval d on market returns
n number of intervals employed
$r_{i,t,d}$ return of stock i at day t with lagging, synchronous and leading interval d
α_i intercept of the regression of stock i
$r_{m(i),t+d}$ return of the respective market index of stock i at day t delayed and leaded by interval d
ε_i regression error term

The Trade-to-Trade Method[38] regresses stock returns against the return of the market on synchronized dates. Since returns only stem from days with trading activity, autocorrelation in the stock returns is avoided. Furthermore, the problem of heteroscedastic residuals (due to the variable intervals causing the variance to behave (approximately) proportional to the length of the residuals) is avoided by scaling the multi-period returns with the length of the non-trading interval between the consecutive trades.

The regression equation is denoted by:

$$\frac{r_{i,t}}{\sqrt{d_{i,t}}} = \alpha_i \frac{1}{\sqrt{d_{i,t}}} + \beta_i^{TTTM} \frac{r_{m(i),t}}{\sqrt{d_{i,t}}} + \varepsilon_i \quad (23)$$

31 Dimson, Risk measurement when shares are subject to infrequent trading, JFE, vol. 7, no. 2 (1979): 197-226; Marsh, Equity Rights Issues and the Efficiency of the UK Stock Market, The Journal of Finance, vol. 34, no. 4 (1979): 839-862; Dimson/Marsh, The stability of UK risk measures and the problem of thin-trading, The Journal of Finance, vol. 38, no. 3 (1983): 753-783.
32 Bartholdy/Riding, op. cit. (footnote No. 27): 241-254.
33 McInish/Wood, op. cit. (footnote No. 3): 277-286.
34 Berglund/Liljeblom/Löflund, op. cit. (footnote No. 27): 41-64.
35 Berglund/Liljeblom/Löflund, op. cit. (footnote No. 27): 41-64.
36 Fowler/Rorke, op. cit. (footnote No. 30): 279-283.
37 Dimson, op. cit. (footnote No. 31): 197-226.
38 Schwert, Stock Exchange Seats and Capital Assets, JFE, vol. 4, no. 1 (1977): 51-78; Franks/Boyles/Hecht, An industry Study of the Profitability of Mergers in the United Kingdom, The Journal of Finance, vol. 32, no. 5 (1977): 1513-1525; Marsh, op. cit. (footnote No. 31): 839-862; Dimson/Marsh, op. cit. (footnote No. 31): 753-783.

β_i^{TTTM} corrected beta estimator of stock i employing the Trade-to-Trade Method and, simultaneously, the regression coefficient on the market return
t days of trading
$r_{i,t}$ non-zero return on stock i at day t
$d_{i,t}$ length of the non-trading interval of stock i between two consecutive trades
α_i intercept of the regression of stock i
$r_{m(i),t}$ return on the respective market of stock i at day t
ε_i regression error term

The Asymptotic Beta Coefficient Method[39] is built upon the Triple Single Factor Model by adding a lagging and a leading beta as well as an autocorrelation coefficient, thus, generalizing it by allowing for price adjustment delays of even more than one or two days. Studies used lagging and leading terms up to ten, however, the higher lagging and leading terms have not been found to be effective. For example, Bradfield/Barr[40] investigated the effect of a variety of lagging and leading terms, recommending at least one lagging and one contemporaneous term if the market proxy is largely free from an information delay. In contrast, if the market proxy is expected to be affected by an information delay, they recommend at least one lagging, one leading and, a contemporaneous term.

The closed-end formula for the beta estimator and the regression equations are denoted by:

$$\beta_i^{ABCM} = \frac{\beta_i + \sum_{j=-1}^{-J}\beta_{i,j} + \sum_{k=1}^{K}\beta_{i,k}}{1 + \sum_{j=-1}^{-J}\beta_{m(i),j} + \sum_{k=1}^{K}\beta_{m(i),k}}$$

with

$$r_{i,t} = \alpha_{i,n} + \beta_{i,n} \cdot r_{m(i),t+n} + \varepsilon_i,\ n \in \{j,k\}$$
$$r_{m(i),t} = \alpha_{m(i),n} + \beta_{m(i),n} \cdot r_{m(i),t+n} + \varepsilon_{m(i)},\ n \in \{j,k\} \quad (24)$$

β_i^{ABCM} corrected beta estimator of stock i employing the Asymptotic Beta Coefficient Method
j number of lagging and k the number of leading intervals on market returns
β_i OLS regressions coefficients on both, stock i and the respective market m of stock i, with j lagging and k leading terms

For the regression equations:

$r_{i,t}$ return of stock i at day t
$r_{m(i),t}$ return of the respective market m of stock i at day t
$r_{m(i),t+n}$ return of the respective market m of stock i with lagging, synchronous or leading interval n
$\alpha_{i,n}$, $\alpha_{m(i),n}$ intercept of the regression of stock i and the respective market, respectively, with lagging, synchronous or leading interval n
ε_i, $\varepsilon_{m(i)}$ regression error terms

The Error Correction Model[41] is built upon the problem in systematic risk estimation that the return of a thinly traded stock is not instantaneously reflected by the market return and, consequently, the market model estimates may be biased.

The Closed-end formula for the beta estimator and the regression equation are denoted by:

$$\beta_i^{ECM} = -\frac{\varphi_i}{\lambda_i}$$

$$\Delta r_{i,t} = \alpha_i + \gamma_i \Delta r_{m(i),t} + \lambda_i r_{i,t-1} + \varphi_i r_{m(i),t-1} + \varepsilon_i$$

with

$$\Delta r_{i,t} = r_{i,t} - r_{i,t-1}$$

$$\Delta r_{m(i),t} = \frac{r_{i,t} - r_{i,t-1}}{r_{m(i),t} - r_{m(i),t-1}} \quad (25)$$

β_i^{ECM} corrected beta estimator on stock i employing the Error Correction Model
$\Delta r_{i,t}$ change in returns of stock i between day t and the day before t-1
$\Delta r_{m(i),t}$ relative change of the returns of stock i between day t ($r_{i,t}$) vand the day before t-1 ($r_{i,t-1}$)and of the returns of the respective market of stock i between day t($r_{m(i),t}$) and the day before t-1($r_{m(i),t-1}$)
á$_i$ intercept of the regression on stock i
γ_i, λ_i, φ_i regression coefficients of the regression on stock i
ε_i regression error term

Returns for each day are computed by the change in the logarithmic price indices $r_{i,t} = \ln(p_{i,t}) - \ln(p_{i,t-1})$, since changes in natural logarithms can be regarded as good approximations for stock returns, assuming continuous compounding.

$r_{i,t}$ return of stockiat day t
$\rho_{i,t}$ price index of stock i at day t
$\rho_{i,t-1}$ price index of stock i the day before t-1

39 Cohen/Hawawini/Maier/Schwartz/Whitcomb, Friction in the trading process and the estimation of systematic risk, JFE, vol. 12, no. 2 (1983): 263-278.
40 Bradfield/Barr, Risk Estimation in the Thinly Traded JSE Environment, South African Journal of Business Management, vol. 20, no. 4 (1989): 169-173.
41 Luoma/Martikainen/Perttunen, Thin trading and estimation of systematic risk: An application of an error-correction model, Annals of Operations Research, vol. 45, no. 1 (1993): 297-305.

Finally, the Adjusted OLS Beta Method[42] is based on the finding that simply adjusting the OLS-Beta estimate for the inverse of the proportion of potential trading days results in an (systematically) unbiased beta estimate. The formula is denoted by:

$$\beta_i^{AOLS} = \beta_i^{OLS} \frac{TD_i}{DT_i} \tag{26}$$

β_i^{AOLS} adjusted OLS beta estimate of stock i
β_i^{OLS} ordinary least squares beta estimate of stock i
TD_i total number of potential trading days of stock i in the total measurement period
DT_i actual number of days stock i is traded in the total measurement period

III. Performance Study: Indicating (Il-)liquidity versus Correcting Betas

1. Previous Research

Both, the performance of the various (il-)liquidity indicators and the beta correction procedures were analyzed. Focusing on (il-)liquidity indicators emphasizing the trading frequency dimension of liquidity, it was generally demonstrated that stocks with high turnover ratios, considered to be more liquid, dictate lower returns as compared to stocks with low turnover ratios.[43] Furthermore, the Turnover Indicator turned out to be the best and the Illiquidity (Amihud-Hasbrouck) Indicator as well as the Liquidity (Amivest-Hasbrouck) Indicator to be good proxies for liquidity, while the Gamma Indicator and the Trading Volume Indicator perform relatively poor.[44] Emphasizing the price impact dimension, the Illiquidity (Amihud) Indicator was concluded to be superior to the Turnover Indicator and the Gamma Indicator[45] and the LM Indicator outperformed other indicators in predicting stock returns.[46]

Examining the performance across beta correction procedures, the overall performance as well as bias and accuracy separately, were evaluated:

i. Emphasizing the overall performance (bias and accuracy) of the corrected beta estimates, the Trade-to-Trade Method generally yielded superior beta estimates as compared to the Repetition of Last Quote Procedure and the Triple Single Factor Model on the U. S. market for different return periodicities, levels of trading days and levels of beta (this result holds especially for a daily return periodicity, but at the expense of poorly eliminating bias, only being eliminated with a weekly and monthly periodicity),[47] as compared to the Repetition of Last Quote and the Uniform Quote Procedure on the Danish market[48] and as compared to the Aggregated Coefficients Model and the Asymptotic Beta Coefficients Method on the British and the South African market.[49]

ii. Emphasizing bias separately, the Trade-to-Trade Method and the Error Correction Model generally yielded least biased estimates for different levels of thinness within a set of ten beta correction techniques, but with varying performance across different trading frequencies.[50] Somewhat similarly, the Trade-to-Trade Method generated least biased beta estimates up to high levels of thin trading, followed by the Adjusted OLS Beta Method.[51] Furthermore, the Aggregated Coefficients Model was concluded to outperform the Triple Single Factor Model for high levels of thinness, but vice versa for relatively thickly traded stocks.[52] Being critical in nature, some studies found evidence that neither the Aggregated Coefficients Model, the Adjusted OLS Beta Method, nor the Triple Single Factor Model adequately control for nonsynchronous trading bias.[53]

iii. Emphasizing accuracy separately, the Trade-to-Trade Method generally performed best across all levels of trading, whereas the Repetition of Last Quote Procedure as well as the Uniform Quote Procedure severely mis-specified the beta estimator.[54]

Furthermore, the performance dimensions turned out to be related, since less bias comes at the expense of a higher standard error. Whereas the Adjusted OLS Beta Method outperforms the Aggregated Coefficients Model

42 Fowler/Rorke, op. cit. (footnote No. 30): 279-283.
43 Datar/Naik/Radcliffe, op. cit. (footnote No. 15): 203-219; Chan/Faff, Asset pricing and the Illiquidity Premium, The Financial Review, vol. 40, no. 4 (2005): 429-458; Nguyen/Mishra/Prakash/Ghosh, op. cit. (footnote No. 16): 379-398; Ho/Hung, Investor sentiment as conditioning information in asset pricing, JBF, vol. 33, no. 5 (2009): 892-903.
44 Wang/Kong, Illiquidity and asset pricing in the Chinese stock market, China Finance Review International, vol. 1, no. 1 (2011): 57-77.
45 Liang/Kong, Empirical test on the pricing of liquidity measures in Chinese stock market, Management Science, vol. 21, no. 3 (2008): 85-93; Goyenko/Holden/Trzcinka, Do liquidity measures measure liquidity? JFE, vol. 92, no. 2 (2009): 153-181.
46 Zhang/Yang/Su/Zhang, op. cit. (footnote No. 7): 168-186.
47 Serra/Martelanc, op. cit. (footnote No. 28): 49-78.
48 Bartholdy/Olson/Peare, Conducting Event Studies on a Small Stock Exchange, The European Journal of Finance, vol. 13, no. 3 (2007): 227-252.
49 Dimson/Marsh, op. cit. (footnote No. 31): 753-783; Bowie/Bradfield, A Review of Systematic Risk Estimation on the JSE, De Ratione, vol. 7, no. 1 (1993): 6-22.
50 Luoma/Martikainen/Perttunen/Pynnonen, op. cit. (footnote No. 24): 471-476.
51 McClelland/Auret/Wright, Thin-Trading and Beta Estimation: Results from a Simulated Environment, Journal of Studies in Economics and Econometrics, vol. 38, no. 2 (2014): 19-31.
52 Dimson, op. cit. (footnote No. 31): 197-226.
53 Fowler/Rorke/Jog, op. cit. (footnote No. 25): 23-32; McInish/Wood, op. cit. (footnote No. 3): 277-286.
54 Maynes/Rumsey, Conducting event studies with thinly traded stocks, JBF, vol. 17, no. 1 (1993): 145-157.

and OLS beta estimates perform worst in terms of bias, OLS beta estimates are most precise in terms of standard errors, and beta estimates corrected employing the Adjusted OLS Beta Method are noisiest.[55]

2. Research Questioning

Since both, (il-)liquidity indicators and beta correction procedures are all exposed to individual strengths and weaknesses, this section examines whether the systematic distortion of betas is better identified using (il-)liquidity indicators (Hypothesis/Null-Hypothesis: The (il-)liquidity indicators better/do not better identify the systematic distortion) or should be eliminated using beta correction procedures (Hypothesis/Null-Hypothesis: The beta correction procedures better/do not better eliminate the systematic distortion).

3. Sample Data and Research Methodology

The analysis is based on stock (closing) price and trading volume data (directly retrieved from the stock exchanges) for the years 2017 and 2018 of stocks being continuously traded and negotiated in nine indices categorized in large and small markets (number of stocks captured in brackets):

- Large markets: Dow Jones Industrial Average (30), Shanghai Stock Exchange 50 Index (50), EuroStoxx 50 (50)
- Small markets: Amsterdam Stock Exchange Index (25), Nordic Stock Exchange 40 Index (40), Austrian Traded Index (20), Czech Traded Index (9), Ljubljana Stock Exchange Index (5), Budapest Stock Exchange Index (11) (with the latter four formerly constituting the Central and Eastern European Stock Exchange Group)

To avoid any distortions caused by currency values, all prices are converted into USD employing daily pro rata exchange rates.

The experimental setting of this study is based on a simulated computer quasi-experiment, estimating betas of incomplete price/return series and comparing them with the underlying beta that generated the simulated data. Since the true (underlying) beta is known a priori (in contrast to reality, where the true beta is not actually known), it can be compared to the estimated beta and, the performance of the (il-)liquidity indicators indicating and the correction procedures directly correcting distortions in beta estimates, can be observed without the impact of environmental noise. The performance is observed by running horseraces among the (il-)liquidity indicators, among the beta correction procedures and, the (il-)liquidity indicators against the beta correction procedures, randomly removing an increasing number of trading days.[56] For each of the 240 stocks considered, one up to 80 trading days are removed from the original return series (thus, introducing a level of thin trading of about 30 percent at maximum), with 100 trading simulations run for each. Aggregated for the years 2017 and 2018, this results in 16,000 trading simulations per stock and 3,840,000 trading simulations across all stocks considered.

For beta estimation, three fundamental methodological decisions are made concerning market index, return history, and return periodicity:

i. Theoretically, according to the CAPM, a broad market index should be employed, whereas, according to the home bias of investors, a local index should be used. Empirical results concerning the impact of the market index on beta estimations are somewhat conflicting, either concluding a relatively low or a high impact on beta estimates, respectively, and concluding a broader market index yielding higher beta estimates. This study follows common practice utilizing the market index the respective stock is negotiated in.

ii. As for the return history, the larger the number of observations, the higher the precision and the smaller the error in the beta estimate, being favored by a long history of returns. There is a tendency for the explanatory power of the regression equation and the mean value of beta to rise as the collection period increases. However, the probability correspondingly increases that the company has suffered structural changes, and, hence, inducing a change in the beta estimate. Since companies do not suffer significant beta changes in less than three years,[57] this study employs a one-year history of returns,[58] resulting in two separate estimates for the years 2017 and 2018.

iii. Prior research concludes that the return periodicity has a significant impact on the beta estimate, but with diverging results concerning the direction of the correlation. While some studies conclude an increase in the return interval to yield a corresponding increase in the beta esti-

55 Sercu/Vandebroek/Vinaimont, op. cit. (footnote No. 26): 1196-1219.

56 This methodology follows Luoma/Martikainen/Perttunen, op. cit. (footnote No. 41): 297-305 as well as Serra/Martelanc, op. cit. (footnote No. 28): 49-78.

57 Daves/Ehrhardt/Kunkel, Estimating systematic risk: the choice of return interval and estimation period, Journal of Financial and Strategic Decisions, vol. 13, no. 1 (2000): 7-13.

58 Along with Zhang/Yang/Su/Zhang, op. cit. (footnote No. 7): 168-186.

mates,[59] others demonstrate that the covariance of the returns does not vary proportionally to the market variance with different return intervals, with the beta estimates of high-risk stocks increasing and the beta estimates of low-risk stocks decreasing with a lengthening in the return interval.[60] Furthermore, betas of stocks with a below market average value were documented to decrease, whereas stocks with an above market average value generally face an increasing beta when switching the return periodicity from daily to monthly.[61] This study employs daily return data for three conceptual reasons:[62] (i) First, it generally produces a lower standard error in beta estimates. (ii) Second, since the beta estimate of thinly traded stocks is systematically downwards biased, the non-synchronization problem between market data and stock data becomes more serious,[63] enabling to maximize the performance measurement of the (il-)liquidity proxies and the beta correction procedures employed. (iii) Finally, all (il-)liquidity indicators employed are conceptually based on daily stock return data.

The performance of both, (il-)liquidity indicators indicating and correction procedures directly correcting distortions in beta estimates, is measured by the Relative Log-scaled Absolute Estimation Error, since it avoids both, an upwards bias and the netting effect of positive and negative deviations, and, hence, considering solely the strength of the deviation.[64] The Relative Log-scaled Absolute Estimation Error is to be minimized. It is denoted by:

$$\mathrm{RLAE}_{i,k} = \ln\left(\widehat{\lambda_{l,k}}\right) - \ln\left(\lambda_{i,k}\right) \tag{27}$$

$\mathrm{RLAE}_{i,k}$ Relative Log-scaled Absolute Estimation Error
$\widehat{\lambda_l}$ estimated value (of (il-)liquidity indicator and beta estimate)
λ_i observed value (of (il-)liquidity indicator and beta estimate)
i index on stocks
k index indicating the number of removed trading days

To ensure comparability of the performance of the (il-)liquidity indicators and the beta correction procedures, the error indicators are adjusted differently. The (il-)liquidity indicators are adjusted in a two-step computation with $\widehat{\lambda_{l,k}}$ itself computed as an error indicator of the relative difference – caused by thin trading – of the change in the (il-)liquidity indicator employed and the change of the (observed) beta estimate in terms of the change in the (observed) beta estimate, denoted by:

$$\widehat{\lambda_{l,k}} = \frac{\phi_{i,k} - \phi_{i,k=0}}{\phi_{i,k=0}}$$

$$\lambda_{i,k} = \frac{\beta_{i,k} - \beta_{i,k=0}}{\beta_{i,k=0}} \tag{28}$$

$\phi_{i,k}$ respective (il-)liquidity indicator of stock
$\beta_{i,k=0}$ (observed) beta estimate of stock
k index indicating the number of removed trading days (with $k=0$ indicating full trading)

The beta correction procedures are adjusted in a one-step computation with $\widehat{\lambda_l}$ indicating the corrected beta estimate resulting from employing the various error correction procedures and λ_i indicating the observed beta estimate, thus resulting in the relative difference between the corrected and the observed beta estimate in terms of the observed beta estimate, denoted by:

59 Breen/Lerner, On the Use of β in Regulatory Proceedings, The Bell Journal of Economics and Management Science, vol. 3, no. 2 (1972): 612-621; Pogue/Solnik, The Market Model Applied to European Common Stocks: Some Empirical Results, JFQA, vol. 9, no. 6 (1974): 917-944; Schwartz/Whitcomb, The Time-Variance Relationship: Evidence on Autocorrelation in Common Stock Returns, The Journal of Finance, vol. 32, no. 1 (1977): 41-55; Smith, The Effect of Intervalling on Estimating Parameters of the Capital Asset Pricing Model, JFQA, vol. 13, no. 2 (1978): 313-332; Corhay, The Intervalling Effect Bias in Beta: A Note, JBF, vol. 16, no. 1 (1992): 61-73; Perron/Chun/Vodounou, Sampling interval and estimated betas: Implications for the presence of transitory components in stock prices, Journal of Empirical Finance, vol. 20, no. 1 (2013): 43-62; Hong/Satchell, The sensitivity of beta to the time horizon when log prices follow an Ornstein-Uhlenbeck process, The European Journal of Finance, vol. 20, no. 3 (2014): 264-290.

60 Handa/Kothari/Wasley, The relation between the return interval and betas: Implication for size effect, JFE, vol. 23, no. 1 (1989): 79-100.

61 Hawawini, Why beta shifts as the return interval changes, Financial Analysts Journal, vol. 39, no. 3 (1983): 73-77.

62 Along with Berglund/Liljeblom/Löflund, op. cit. (footnote No. 27): 41-64; Daves/Ehrhardt/Kunkel, op. cit. (footnote No. 57): 7-13; Davidson/Josev, op. cit. (footnote No. 27): 111-117; Bartholdy/Olson/Peare, op. cit. (footnote No. 48): 227-252; Hasbrouck, op. cit. (footnote No. 9): 1445-1477.

63 Scholes/Williams, op. cit. (footnote No. 29): 309-327.

64 The Relative Log-scaled Absolute Estimation Error is utilized in valuation research to evaluate accuracy and bias of valuation multiples e.g. by Kaplan/Ruback, The Valuation of Cash Flow Forecasts: An Empirical Analysis, The Journal of Finance, vol. 50, no. 4 (1995): 1059-1093; Kim/Ritter, Valuing IPOs, JFE, vol. 53, no. 3 (1999): 409-437; Gilson/Hotchkiss/Ruback, Valuation of Bankrupt Firms, The Review of Financial Studies, vol. 13, no. 1 (2000): 43-74; Lie/Lie, Multiples used to Estimate Corporate Value, Financial Analysts Journal, vol. 58, no. 2 (2002): 44-54; Herrmann/Richter, Pricing with Performance-Controlled Multiples, SBR, vol. 55 (2003): 194-219; Sommer/Rose/Wöhrmann, Negative Value Indicators in Relative Valuation – An Empirical Perspective, JBVELA, vol. 9, no. 1 (2014): 23-54.

$$\widehat{\lambda_{1,k}} = \beta^{c}_{i,k}$$
$$\lambda_{i,k} = \beta_{i,k=0} \quad (29)$$

$\beta^{c}_{i,k}$ corrected beta estimate of stock i
$\beta_{i,k}$ (observed) beta estimate of stock i
k index indicating the number of removed trading days (with k=0 indicating full trading)

To arrive at a one-dimensional error indicator, the various error indicators computed across all stocks and across different levels of thin trading are aggregated employing the median to ensure unbiased estimates on accuracy (for large samples).[65]

4. Results – Performance according to Market Segments

Table 2 reports the results on performance (accuracy) employing RLAE with the superior (il-)liquidity indicators/beta correction procedures indicated as follows: The superior (il-)liquidity indicator as well as the superior beta correction procedure are indicated in bold italics, the comprehensively superior indicator/correction procedure is indicated in bold italics and additionally underlined.

Since the accuracy of historical betas improves with portfolio size, the results are reported separately for the total sample (total market) as well as for small and large markets.[66] The results aggregate one to 80 trading days removed (thus, introducing a level of thin trading of about 30 percent at maximum). For the Aggregated Coefficients Model, the Asymptotic Beta Coefficient Method and the Error Correction Model, "+" indicates leading terms and "–" indicates lagging terms employed. The results on the test statistic of the two-tailed Wilcoxon Signed-Rank Test (employing the normal approximation for large samples) indicate whether the error indicators for the best performing (il-)liquidity indicator and the best performing beta correction procedure differ. The respective z-values are reported without brackets, p-values in brackets.

The results indicate the (il-)liquidity indicators to (statistically significantly) outperform beta correction procedures across all market segments (with the best performing beta correction procedures outperformed by at least three (il-)liquidity indicators), but with varying (il-)liquidity indictors dominating. For the total market, the Return-to-Turnover Indicator best indicates beta distortion, whereas for large markets, the Illiquidity (Amihud-Hasbrouck) Indicator and, for small markets, the Liquidity (Amivest-Hasbrouck) Indicator best indicate beta distortions. Additionally, the Illiquidity (Amihud) Indicator generates reasonable indications. Among the beta correction procedures, the Trade-to-Trade Method (for the total as well as for large markets) and the Repetition of Last Quote Procedure (for small markets) best correct beta estimates. Additionally, the Uniform Returns Procedure and the Uniform Quotes Procedure generate reasonable beta estimates. Therefore, valuation analysts should better select (peers') betas by examining (il-)liquidity indicators than correcting them employing beta correction procedures.

5. Results – Performance according to Level of Thin Trading

Since beta adjustments perform differently across differently traded markets,[67] table 3 reports the results on the performance (accuracy) employing RLAE of the (il-)liquidity indicators and the beta correction procedures for varying levels of thin trading, with the sample sorted in ascending order in steps of 10 trading days removed in the range of one to 80 trading days (thus, introducing a level of thin trading of about 30 percent at maximum). The sample comprises total market data. The superior (il-)liquidity indicator as well as the superior beta correction procedure are indicated in bold italics, the comprehensively superior indicator/correction procedure is indicated in bold italics and additionally underlined. For the Aggregated Coefficients Model, the Asymptotic Beta Coefficient Method and the Error Correction Model, "+" indicates leading terms and "–" indicates lagging terms employed. The results on the test statistic of the two-tailed Wilcoxon Signed-Rank Test (employing the normal approximation for large samples) indicate whether the error indicators for the best performing (il-)liquidity indicator and the best performing beta correction procedure differ. The respective z-values are reported without brackets, p-values in brackets.

The results indicate a clear segmentation between small and higher levels of thin trading. Whereas for small levels of thin trading (up to 20 days of non-trading) the simple beta correction procedures generate more accurate (statistically significant) beta estimates, they are (again statistically significantly) outperformed by the (il-)liquidity indicators for higher levels of thin trading (starting with 21 up to 80 days of non-trading). For a low level of thin trading, the Uniform Quotes Procedure and the Repetition of Last Quote Procedure generate the most accurate beta estimates, closely followed by (il-)liquidity indicators (i. e., Turnover Indicator and Return-to-Turnover Indicator). Additionally, the Uniform Returns Procedure and the Adjusted OLS Beta Method generate reasonable beta estimates. Among the (il-)liquidity indicators, the two Trading Volume Indicators as well as the Zero Volume Indicator addi-

65 Along with Chullen/Kaltenbrunner/Schwetzler, Does consistency improve accuracy in multiple-based valuation?, JBE, no. 6 (2015): 635-662. The harmonic mean is biased downwards by about as much as the arithmetic mean is biased upwards; Dittmann/Maug, Biases and Error Measures: How to Compare Valuation Methods, Working Paper (2008): 1-39.

66 Mantripragada, Beta Adjustment Methods. Journal of Business Research, vol. 8, no. 3 (1980): 329-339.

67 Luoma/Martikainen/Perttunen/Pynnonen, op. cit. (footnote No. 24): 471-476.

Table 2: Performance (RLAE) according to market segments

(Il-)liquidity indicators Beta correction procedures		Total market	Large markets	Small markets
(Il-)liquidity indicators:				
Illiquidity (Amihud) Indicator		0.883	0.885	0.882
Illiquidity (Amihud-Hasbrouck) Indicator		0.919	<u>0.843</u>	1.002
Adjusted Illiquidity (Amihud) Indicator		1.068	1.209	0.929
Liquidity (Amivest) Indicator		1.223	1.370	1.070
Liquidity (Amivest-Hasbrouck) Indicator		0.906	0.951	<u>0.861</u>
Log Liquidity (Amivest) Indicator		1.310	1.244	1.384
Return-to-Turnover Indicator		<u>0.874</u>	0.881	0.866
Turnover Indicator		1.035	1.172	0.901
Monetary Trading Volume Indicator		1.035	1.162	0.912
Quantity Trading Volume Indicator		1.031	1.165	0.901
Gamma Indicator		1.218	1.158	1.290
LM Indicator		5.957	6.396	5.422
Zero Returns Indicator		1.101	1.249	0.952
Zero Volume Indicator		1.068	1.209	0.929
Extended Zero Returns Indicator		4.066	4.714	3.105
Extended Zero Volume Indicator		4.104	4.765	3.099
Beta correction procedures:				
Repetition of Last Quote Procedure		1.041	1.172	0.910
Uniform Quotes Procedure		1.109	1.231	0.975
Uniform Returns Procedure		1.104	1.228	0.975
Triple Singe Factor Model		1.631	1.607	1.655
Generalized Single Factor Model		2.659	2.905	2.374
Aggregated Coefficients	+1 / -1	1.451	1.519	1.376
Model	+1 / -2	1.658	1.727	1.582
	+1 / -3	1.738	1.820	1.652
	+1 / -4	1.812	1.891	1.732
	+1 / -5	1.879	1.959	1.796
Trade-to-Trade Method		**0.938**	**0.929**	0.947
Asymptotic Beta	-1	1.290	1.233	1.348
Coefficient Method	-2	1.499	1.458	1.540
	-3	1.592	1.543	1.643
	-4	1.852	1.794	1.910
	-5	1.978	1.947	2.010
Error Correction Model	-1	2.690	2.896	2.460
	-2	2.886	3.098	2.645
	-3	2.920	3.105	2.715
	-4	2.978	3.196	2.734
	-5	2.941	3.180	2.675
Adjusted OLS Beta Method		1.207	1.371	1.034
Wilcoxon Signed-Rank Test:		-245.40 (0.0000)	-240.31 (0.0000)	

tionally generate reasonable indications on beta distortion. For a high level of thin trading, the Return-to-Turnover Indicator (for medium levels of thin trading starting with 21 up to 50 days of non-trading) and the Illiquidity (Amihud-Hasbrouck) Indicator (for the highest levels of thin trading starting with 51 up to 80 days of non-trading) dominate. Additionally, the Illiquidity (Amihud) Indicator and the Liquidity (Amivest-Hasbrouck) Indicator generate reasonable indications on beta distortion across all levels of thin trading. The superior beta correction procedure is outperformed by at least three (il-)liquidity indicators, allowing for the general conclusion on superiority of the (il-)liquidity indicators. Among the beta correction procedures, the Trade-to-Trade Method clearly dominates all other beta correction procedures across all levels of (higher) thin trading. Additionally, the Repetition of Last Quote Procedure, the Uniform Returns Procedure, the Uniform Quotes Procedure and the Asymptotic Beta Coefficient Method (with the latter increasing performance with higher levels of thin trading) generate reasonable beta estimates.

As compared to the results for the market segmentation, there appear some similarities, but also some substantial inconsistencies. For the higher levels of thin trading, the results generally coincide, with the (il-)liquidity indicators outperforming the beta correction procedures. Coinciding with the results drawn for the

Table 3: Performance (RLAE) according to level of thin trading

Level of thin trading		1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
(Il-)liquidity indicators:									
Illiquidity (Amihud) Indicator		1.081	0.880	0.848	0.844	0.853	0.859	0.870	0.867
Illiquidity (Amihud-Hasbrouck) Indicator		1.364	1.051	0.938	0.880	0.842	0.823	0.807	0.793
Adjusted Illiquidity (Amihud) Indicator		0.837	0.915	1.024	1.080	1.141	1.180	1.192	1.203
Liquidity (Amivest) Indicator		1.110	1.010	1.153	1.200	1.262	1.310	1.309	1.316
Liquidity (Amivest-Hasbrouck) Indicator		1.098	0.902	0.865	0.860	0.878	0.886	0.891	0.891
Log Liquidity (Amivest) Indicator		1.981	1.559	1.395	1.299	1.215	1.144	1.107	1.068
Return-to-Turnover Indicator		1.082	**0.874**	0.838	0.833	0.840	0.844	0.859	0.858
Turnover Indicator		**0.820**	0.880	0.975	1.037	1.107	1.145	1.164	1.177
Monetary Trading Volume Indicator		0.822	0.879	0.975	1.035	1.107	1.148	1.165	1.178
Quantity Trading Volume Indicator		0.825	0.878	0.970	1.029	1.104	1.145	1.163	1.178
Gamma Indicator		1.313	1.252	1.226	1.195	1.210	1.194	1.186	1.180
LM Indicator		5.401	5.752	5.899	5.996	6.077	6.136	6.166	6.184
Zero Returns Indicator		0.831	0.903	1.021	1.091	1.175	1.237	1.278	1.312
Zero Volume Indicator		0.827	0.880	0.991	1.058	1.134	1.193	1.235	1.268
Extended Zero Returns Indicator		4.669	4.324	4.137	4.042	3.944	3.865	3.826	3.779
Extended Zero Volume Indicator		4.733	4.374	4.182	4.084	3.974	3.896	3.850	3.800
Beta correction procedures:									
Repetition of Last Quote Procedure		0.844	0.871	0.975	1.039	1.109	1.141	1.169	1.186
Uniform Quotes Procedure		0.779	0.955	1.063	1.138	1.192	1.215	1.245	1.259
Uniform Returns Procedure		0.799	0.953	1.058	1.133	1.188	1.210	1.239	1.252
Triple Singe Factor Model		2.341	1.729	1.616	1.547	1.526	1.505	1.491	1.464
Generalized Single Factor Model		4.203	3.318	2.905	2.665	2.418	2.253	2.025	1.807
Aggregated Coefficients	+1 / -1	2.096	1.503	1.406	1.361	1.340	1.362	1.344	1.349
Model	+1 / -2	2.398	1.733	1.580	1.523	1.527	1.550	1.556	1.569
	+1 / -3	2.523	1.839	1.668	1.605	1.586	1.609	1.622	1.634
	+1 / -4	2.657	1.951	1.753	1.672	1.650	1.663	1.671	1.680
	+1 / -5	2.762	2.048	1.836	1.749	1.709	1.713	1.718	1.720
Trade-to-Trade Method		1.018	0.977	**0.972**	**0.955**	**0.924**	**0.897**	**0.885**	**0.870**
Asymptotic Beta	-1	2.287	1.559	1.340	1.227	1.137	1.080	1.033	0.995
Coefficient Method	-2	2.613	1.846	1.585	1.438	1.326	1.252	1.173	1.117
	-3	2.698	1.926	1.678	1.536	1.423	1.341	1.270	1.209
	-4	3.004	2.209	1.954	1.779	1.654	1.583	1.490	1.422
	-5	3.183	2.371	2.077	1.908	1.769	1.701	1.584	1.524
Error Correction Model	-1	4.241	3.319	2.904	2.669	2.435	2.295	2.084	1.894
	-2	4.287	3.424	3.060	2.856	2.656	2.533	2.345	2.184
	-3	4.297	3.436	3.075	2.874	2.693	2.577	2.400	2.239
	-4	4.347	3.485	3.128	2.933	2.745	2.647	2.469	2.325
	-5	4.284	3.433	3.077	2.891	2.443	2.613	2.450	2.296
Adjusted OLS Beta Method		0.838	0.940	1.085	1.189	1.286	1.358	1.443	1.513
Wilcoxon Signed-Rank Test:		-94.19 (0.0000)	-75.20 (0.0000)	-97.40 (0.0000)	-93.22 (0.0000)	-88.64 (0.0000)	-96.68 (0.0000)	-96.63 (0.0000)	-96.51 (0.0000)

total market, the Return-to-Turnover Indicator dominates for the medium levels of thin trading. Coinciding with the results drawn for large markets, the Illiquidity (Amihud-Hasbrouck) Indicator dominates for the highest levels of thin trading. In contrast, the Liquidity (Amivest-Hasbrouck) Indicator dominating in small markets does not dominate the other (il-)liquidity indicators in any level of thin trading. Furthermore, in contrast to the results drawn for the market segmentation, for the very low levels of thin trading, the dominance of the (il-)liquidity indicators cannot be confirmed. These results allow for the general conclusion that valuation analysts should draw their attention to the level of thin trading of (peer's) betas, since the methods dominating vary with the level of thinness. Whereas for low levels of thin trading beta correction procedures should be employed, (il-)liquidity indicators should be favored for medium and high levels of thin trading.

6. Results – Performance according to Level of Risk

Since beta adjustments perform differently for varying levels of risk,[68] table 4 reports the results on the performance (accuracy) employing RLAE of the (il-)liquidity indicators

68 Bartholdy/Riding, op. cit. (footnote No. 27): 241-254; Serra/Martelanc, op. cit. (footnote No. 28): 49-78.

and the beta correction procedures for varying levels of risk (OLS beta magnitudes), with the sample divided into deciles according to the observed fully traded OLS betas sorted in ascending order from the lowest decile (D1) to the highest (D10). The sample comprises total market data. The superior (il-)liquidity indicator as well as the superior beta correction procedure are indicated in bold italics, the comprehensively superior indicator/correction procedure is indicated in bold italics and additionally underlined. For the Aggregated Coefficients Model, the Asymptotic Beta Coefficient Method and the Error Correction Model, "+" indicates leading terms and "–" indicates lagging terms employed. The results on the test statistic of the two-tailed Wilcoxon Signed-Rank Test (using the normal approximation for large samples) indicate whether the error indicators for the best performing (il-)liquidity indicator and the best performing beta correction procedure differ. The respective z-values are reported without brackets, p-values in brackets.

Somewhat coinciding with the results on varying levels of thin trading, the results indicate a clear (statistically significant) superiority of the (il-)liquidity indicators across all levels of risk except for the lowest-risk decile. For the lowest-risk decile, the Uniform Returns Procedure (followed by the Uniform Quotes Procedure and the Adjusted OLS Beta Method) outperform (again statistically significant) the (il-)liquidity indicators. For all other risk deciles, the (il-)liquidity indicators outperform the beta correction procedures, but with varying indicators dominating (i. e., the Illiquidity (Amihud-Hasbrouck) Indicator in the highest-risk deciles 7 to 10, the Return-to-Turnover Indicator in the medium-risk deciles 4 to 6 and, the Liquidity (Amivest-Hasbrouck) Indicator in decile 3 and, the Quantity Trading Volume Indicator to perform best in decile 2). Additionally, the Illiquidity (Amihud) Indicator and, to some extent, the Gamma Indicator (for the higher levels of risk), and the Zero Returns Indicator (best performing (il-)liquidity indicator in the lowest-risk decile) as well as the Trading Volume Indicators, the Turnover Indicator and the Adjusted Illiquidity (Amihud) Indicator (for lower levels of risk), generate reasonable indications on beta distortion. Among the beta correction procedures, the results indicate superiority of the Trade-to-Trade Method for the higher levels of risk and the Repetition of Last Quote Procedure as well as the Uniform Returns Procedure for lower risk levels. Additionally, across all levels of risk, the Uniform Quotes Procedure and the Asymptotic Beta Coefficients Method (employing one lagging term) generate reasonable beta estimates. Disregarding the anomalous superiority of the beta correction procedures in the lowest risk decile, the best performing beta correction procedure is outperformed by at least four (il-)liquidity indicators (except in the highest risk decile), indicating the latter to be strongly superior.

As compared to the results for the market segmentation, there again appear some similarities, but also some substantial inconsistencies. Except for the lowest-risk decile, the results generally coincide with (i) the (il-)liquidity indicators outperforming the beta correction procedures, (ii) the results concluded for total market data, the Return-to Turnover Indicator dominating in the medium-risk deciles, (iii) the results concluded for large markets, the Illiquidity (Amihud-Hasbrouck) Indicator dominating in the high-risk deciles and, (iv) the results concluded for small markets, the Liquidity (Amivest-Hasbrouck) Indicator dominating in risk-decile three. In contrast, the smallest risk deciles one and two show no similarities with those of the market segmentation. The results allow for the general conclusion that valuation analysts should also draw their attention to the level of risk of (peer's) betas, since, although indicating the (il-)liquidity indicators to clearly dominate the beta correction procedures, the dominating indicator varies with the level of risk.

IV. Conclusion

Estimates on betas themselves as well as on peer group betas valuing private firms may be distorted by thin trading effects. Therefore, valuation analysts need to avoid employing distorted beta estimates by either eliminating low liquidity stocks or by directly correcting beta estimates. This article provides an overview of comprehensive sets of 16 popular (il-)liquidity indicators and 10 popular beta correction procedures. Furthermore, it provides a comparison on the performance (according to accuracy) of the (il-)liquidity indicators and the beta correction procedures among themselves as well as against each other.

The results[69] allow for the overall conclusion that the (il-)liquidity indicators better indicate beta distortions caused by thin trading than the beta correction procedures do in eliminating them. This holds for (i) small as well as large stock markets, (ii) across different levels of thin trading and, (iii) across different levels of risk (beta magnitudes). Notable exemptions are stocks experiencing very low levels of thin trading as well as very small-risk stocks. Therefore, valuation analysts should emphasize on employing (il-)liquidity indicators to indicate beta distortions rather than correcting betas employing beta correction procedures. Furthermore, both, the level of thin trading and the level of risk of the underlying stock must be considered. Emphasizing on the superiority of specific (il-)liquidity indicators/error correction procedures, the results generally indicate the Illiquidity (Amihud-Hasbrouck) Indicator and the Return-to-Turnover Indicator as well as the Trade-to-Trade Method to dominate.

69 For the results of similar studies in the Mean-reversion and the Bayesian framework see Grbenic, Peer Group Beta Reliability under Thin Trading Conditions: Results from a Simulated Environment in the Standard, Mean-Reversion and, Bayesian Framework, Working Paper, SSRN (2021): 1-63.

Table 4: Performance (RLAE) according to level of risk

Level of risk		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
(Il-)liquidity indicators:											
Illiquidity (Amihud) Indicator		1.236	0.792	0.835	0.795	0.822	0.804	0.903	0.898	0.873	0.957
Illiquidity (Amihud-Hasbrouck) Indicator		1.703	1.100	1.025	0.859	0.870	0.823	0.821	0.795	0.752	0.727
Adjusted Illiquidity (Amihud) Indicator		0.898	0.711	0.830	0.987	0.989	1.072	1.184	1.248	1.343	1.447
Liquidity (Amivest) Indicator		0.943	0.839	1.006	1.130	1.172	1.240	1.351	1.441	1.471	1.609
Liquidity (Amivest-Hasbrouck) Indicator		1.168	0.799	0.800	0.773	0.828	0.835	0.935	0.974	0.939	1.075
Log Liquidity (Amivest) Indicator		2.259	1.629	1.480	1.331	1.265	1.200	1.117	1.132	1.098	1.001
Return-to-Turnover Indicator		1.238	0.804	0.838	0.752	0.802	0.802	0.872	0.900	0.864	0.958
Turnover Indicator		0.920	0.693	0.807	0.935	0.959	1.025	1.143	1.222	1.264	1.403
Monetary Trading Volume Indicator		0.921	0.689	0.809	0.931	0.968	1.026	1.143	1.191	1.291	1.402
Quantity Trading Volume Indicator		0.920	0.688	0.818	0.925	0.973	1.025	1.138	1.192	1.266	1.403
Gamma Indicator		2.449	1.304	1.513	1.069	1.118	1.033	1.158	0.959	1.084	0.931
LM Indicator		3.961	5.105	5.581	5.870	5.877	6.191	6.255	6.233	6.759	6.855
Zero Returns Indicator		**0.889**	0.733	0.857	1.028	1.028	1.114	1.208	1.287	1.400	1.492
Zero Volume Indicator		0.931	0.716	0.827	0.985	0.978	1.047	1.161	1.234	1.371	1.462
Extended Zero Returns Indicator		3.799	3.454	4.729	4.152	4.118	3.978	3.824	3.689	4.148	4.389
Extended Zero Volume Indicator		3.984	3.534	4.809	4.205	4.142	3.995	3.803	3.736	4.173	4.409
Beta correction procedures:											
Repetition of Last Quote Procedure		0.893	**0.746**	**0.837**	0.997	0.988	1.036	1.108	1.169	1.257	1.382
Uniform Quotes Procedure		0.706	0.848	0.954	1.106	1.078	1.122	1.216	1.237	1.320	1.436
Uniform Returns Procedure		0.699	0.846	0.951	1.101	1.070	1.115	1.195	1.236	1.317	1.431
Triple Singe Factor Model		1.729	1.451	1.559	1.524	1.602	1.706	1.660	1.698	1.637	1.730
Generalized Single Factor Model		1.729	2.090	2.320	2.673	2.472	2.648	2.851	2.968	3.076	3.225
Aggregated Coefficients Model	+1 / -1	1.505	1.284	1.338	1.437	1.500	1.565	1.430	1.531	1.442	1.473
	+1 / -2	1.805	1.487	1.651	1.527	1.758	1.775	1.664	1.703	1.643	1.572
	+1 / -3	1.868	1.687	1.731	1.532	1.867	1.852	1.759	1.769	1.670	1.669
	+1 / -4	2.047	1.776	1.776	1.579	1.901	1.915	1.907	1.768	1.730	1.743
	+1 / -5	2.034	1.845	1.882	1.711	1.928	2.028	1.960	1.851	1.778	1.775
Trade-to-Trade Method		0.949	0.935	0.912	**0.973**	**0.987**	**0.908**	**0.936**	**0.931**	**0.942**	**0.903**
Asymptotic Beta Coefficient Method	-1	1.557	1.299	1.274	1.287	1.293	1.312	1.215	1.260	1.251	1.177
	-2	1.852	1.614	1.493	1.396	1.491	1.516	1.382	1.477	1.405	1.398
	-3	1.880	1.734	1.488	1.552	1.650	1.623	1.522	1.558	1.485	1.479
	-4	2.211	1.999	1.777	1.758	1.832	1.899	1.810	1.777	1.820	1.668
	-5	2.786	2.191	2.010	1.895	1.823	2.008	1.795	1.813	1.784	1.856
Error Correction Model	-1	1.639	2.158	2.476	2.651	2.597	2.783	2.905	2.997	3.107	3.253
	-2	1.775	2.323	2.553	2.892	2.801	3.011	3.131	3.206	3.299	3.456
	-3	1.895	2.409	2.641	2.895	2.837	3.013	3.144	3.236	3.318	3.425
	-4	1.713	2.421	2.745	3.002	2.900	3.112	3.178	3.304	3.396	3.505
	-5	1.904	2.322	2.701	2.905	2.885	3.076	3.132	3.200	3.342	3.503
Adjusted OLS Beta Method		0.870	0.835	0.965	1.164	1.145	1.238	1.325	1.409	1.474	1.619
Wilcoxon Signed-Rank Test:		-97.0 (0.000)	-89.9 (0.000)	-107.8 (0.000)	-93.3 (0.000)	-88.5 (0.000)	-77.2 (0.000)	-86.4 (0.000)	-87.5 (0.000)	-97.1 (0.000)	-91.3 (0.000)

The results are computed employing daily return data to maximize the indication of the performance measurement, since the non-synchronization problem between market data and stock data becomes more serious. The results are expected to be overall consistent employing weekly or monthly data, but with lower indications on differences in performance, since longer intervals tend to eradicate the distortions in betas caused by thin trading.[70]

The results of this study are restricted to at least three limitations. First, the stocks negotiated particularly in the small indices are distorted by thin trading effects even in case of continuous trading (analysis unreported). Therefore, the estimation errors computed may not indicate full beta distortion and, thus, may impact the results on superiority. Second, when randomly removing trading days from the price/return series of stocks, the respective index price/return series are not adjusted correspondingly. This might again (slightly) modify the results. Finally, the level of thin trading introduced is restricted to a maximum of about 30 percent. Therefore, the results may not be representative for stocks experiencing higher levels of thin trading. ◆

70 Therefore, research on beta measurement etc. regularly employs monthly or weekly return series data to avoid distortions in the results due to thin trading.

Interview:
Alexander Aronsohn on IVS Exposure Draft (2023)

Alexander Aronsohn, FRICS
is a Chartered Surveyor with long and wide-ranging experience encompassing residential asset management, commercial and residential development, rating, national and international valuation and investment.
Alexander is the IVSC Director of Technical Standards for Tangible Assets and assists the IVSC Standards Review Board and its Asset Boards on drafting IVS, perspectives papers, presentations, training and articles.

EBVM: IVSC announced a 3-month public consultation on the proposed changes to International Valuation Standards (IVS) until 28 July 2023. Could you please explain the overall objective of the proposed changes?

Alexander: As part of the IVSC's normal processes, the IVS are reviewed on a two-yearly basis by the IVSC's Standards Review Board and Technical Boards (the Boards) to determine whether any part of the IVS would benefit from amendments or updates.

In performing its review and developing proposed changes to the IVS, the Boards were cognisant of the fact that existing IVS are widely used. The overall objective of the proposed changes is to improve clarity and usefulness of IVS for all stakeholders to build confidence and public trust in valuation.

In making these changes the Board considered a broad range of changes including:

1. ongoing changes in global markets and global valuation, including the increased use of technology and the abundance of available data sources,
2. increased use of specialists and service providers by valuers in the performance of valuations,
3. increased demand by stakeholders, including financial institutions, investors, and regulators, for clarity related to valuation process and the management of valuation risk,
4. additional demands on valuation professionals to address new types of assets or liabilities and to expand the application of valuations into areas such as environmental, social and governance (ESG), and
5. integration of the proposed new financial instruments content.

EBVM: What are the most important changes for you?

Alexander: From my perspective all the changes are important as they have been made to meet a market need.

However, the most important revisions to the general standards are as follows:

1. Adoption of a structure that better aligns with the valuation process and relates to all asset classes (*business valuation, intangible assets, tangible assets and financial instruments*).
2. Additions or expansions to the requirements for data and inputs, valuation models, quality controls, and documentation to reflect the increased complexities of valuations.

3. Inclusion of requirements in relation to the consideration of ESG within the valuation process.
4. Clarification of the roles and responsibilities of the parties involved in valuation, such as service organisations and specialists.
5. Substantial revisions and enhancements to IVS 500 Financial Instruments.

EBVM: What new types of assets or liabilities are referred to in the proposed changes?

Alexander: There are no new types of assets or liabilities particularly referred to in the proposed changes as IVS is an overarching standard that relates to all assets and liabilities. However, having said this the nomenclature of IVS 300 Has been changed to include infrastructure as not all users realised that this standard also referred to infrastructure. Furthermore IVS 400 Real property has been revised to provide further references to agricultural land.

In addition, IVS 500 Financial Instruments has been significantly revised to provide greater clarity on the valuation of financial instruments.

Moreover though there are no new types of assets and liabilities are specifically included in the standard the methodology of valuation is ever changing and to meet market needs the revisions to IVS include a new standard on data and inputs (*IVS 104 Data and Inputs*) and a new standard on the valuation of models (*IVS 105 Valuation Models*).

EBVM: Where has the current discussion about the increasing use of technology and data found expression in the proposed changes?

Alexander: The increasing use of data and technology has been reflected in the addition of the following two new standards:

- IVS 104 Data and Inputs
- IVS 105 Valuation Models

IVS 104 provides the following characteristics of suitable data and models:

- *Accurate*: data and inputs are free from error and bias and reflect the characteristics that they are designed to measure,
- *Appropriate*: data and inputs are relevant for the asset or liability being valued,
- *Complete*: set of data and inputs are sufficient to address attributes of the assets or liabilities,
- *Observable*: data and inputs are obtainable and visible to multiple users or market participants,
- *Timely*: data and inputs reflect the market conditions as of the valuation date,
- *Transparent*: the source of the data and inputs can be traced from their origin.

The aim of the valuation is to maximise as many of these characteristics as possible but at the end of the day the valuer can only use the data that is available and "*If selected data and inputs do not meet all of the characteristics of suitable data and inputs, the data and inputs may still be used as long as the selection is clearly justified and documented*." (IVS 104 -40.2)

IVS 105 provides standards on the use of a service organisation, Valuation Model selection, use and documentation. The standard also highlights the following characteristics of suitable models:

- *Accuracy*: the valuation model is free from error and functions in a manner consistent with the objectives of the valuation,
- *Appropriateness*: the valuation model is suitable for the asset and/or liability being valued, given market conditions at the valuation date,
- *Completeness*: the valuation model addresses all the features of the asset and/or liability to determine value,
- *Timeliness*: the valuation model reflects the market conditions as of the valuation date,
- *Transparency*: all persons preparing and relying on the valuation model must understand how the valuation model works and its inherent limitations.

As with IVS 104 the aim of the valuation is to maximise as many of these characteristics as possible but at the end of the day "*If a chosen model does not meet all these characteristics above, the model may still be compliant so long as the selection is clearly justified and documented*." (IVS 105 – 40.2)

EBVM: Could you please provide some background information on the IVSC perspective on ESG factors & valuation and where those factors have been taken into account in the changes?

Alexander: IVS have always included the requirement for the quantification of ESG's within valuation, but this has always been on an implicit basis. In the past few years the explicit quantification of ESGs within the valuation process across all specialisms (business valuation, financial instruments, intangible assets and tangible assets) has gained even greater prominence across all markets.

As a result the explicit quantification of ESG's within the valuation process has become a key topic for the IVSC as not only is it in the global public interest but also it meets a market need particularly as more standards and regulations are incorporating specific reporting requirements for the quantification of ESGs within valuation.

IVS 101 now includes the following specific requirement in relation to ESG:

"*Environmental, Social and Governance: Any requirements in relation to the consideration of environmental, social and governance factors should be included in the scope of work.*" (IVS 101- 20.2l)

IVS 103 Valuation Approaches also includes the following specific requirements in relation to ESG:

"The terminal value should consider:

(g) Risks and opportunities associated with environmental, social, and governance characteristics of the subject asset. (A20.2)"

"Economic obsolescence may arise when external factors affect an individual asset, or all the assets employed in a business and should be deducted after physical deterioration and functional obsolescence. For real estate, examples of economic obsolescence include:

(e) Adverse changes in the environmental, social, and governance characteristics of the subject asset. (A30.21.)"

IVS 106 Documentation and reporting has also been revised to include the following with ESG requirement within the valuation report:

"Valuation reports must convey the following, at a minimum: (m) environmental, social and governance inputs used and considered, (1vs 106 – 30.6m)"

In addition, IVS 104 Data and Inputs includes a new Appendix on Data and Inputs related to Environmental, Social and Governance factors.

However, IVS are aware that the requirement to consider ESG within valuations is a fast moving field and new requirements are coming into the EU via the EU taxonomy and the forthcoming Internationals Sustainability Standards Board requirements for financial reporting.

The IVSC, as much as it would like, cannot be aware of all changes in relation to ESG every market and there *"The valuer must be aware of relevant legislation and frameworks in relation to the environmental, social and governance factors within their valuation(s)."*

The IVSC is also aware that ESG comprises a mix of qualitative and quantitative factors and therefore not all ESG can be quantified though they can be considered within the valuation process.

Furthermore, the IVSC recognises that in respect of ESG not all the data is currently available, and many markets are still in the early stages of gathering the necessary data.

Therefore, the latest version of IVS has added the following qualifying statements in relation to ESG:

- *10.7 All known or readily available ESG information which would affect how a market participant would assess the value of an asset(s) and what they would pay for an asset should be included in each valuation.*
- *A10.8. ESG factors and the ESG regulatory environment should be considered in valuations to the extent that they are measurable and would be considered reasonable by a peer applying professional judgement.*

EBVM: Why are ESG-aspects taken into consideration in the General Standards but not specifically in the Business Valuation Asset Standard? Do you think the General Standards can cover all business-valuation-relevant ESG-aspects in necessary depth?

Alexander: The IVS General; Standards reply to all Assets and therefore the requirements contained within the IVS General Standards also apply to the Business Valuation Asset Standards (IVS 200, IVS 210, IVS 229 and IVS 230).

When the IVSC Business Valuation Board were reviewing the business valuation standards they found that IVSD 200 to IVS 230 inclusive:

- effectively represent current international best practice; and
- are congruent with the proposed changes in other sections of IVS.

Furthermore, since the adoption and implementation of these standards are at critical junctures in several key jurisdictions, the Business Valuation Board have chosen to not make any substantial changes to these chapters.

However, the point of any consultation is to get market feedback and ensure the revised standards meet market needs so if you do not feel that *"the General Standards relevant can cover all business-valuation-relevant ESG-aspects in necessary depth"* then please participate on the consultation process and advise us accordingly.

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Comments can be submitted in the following ways:

- by personal letter or email to: aaronsohn@ivsc.org
- online through the IVSC's online consultation platform

EBVM: Has the impact of modern artificial intelligence tools and the way business valuators shall deal with such tools found a way into the exposure draft?

Alexander: To a certain extent the impact of modern artificial intelligence tools and the way business valuators will deal with such tools has found its way in the IVS Exposure Draft through the inclusion of IVS 104 Data and Inputs and IVS 105 Valuation Models.

However, from a valuation perspective the role of the IVS is to provide valuation standards to meet current market requirements and not to lead the market.

The IVS Boards noted that though the use of artificial intelligence in valuations is a fast-developing field it is still in the early stages and therefore the IVSC Boards did not feel that specific standards in relation to this were relevant at this point in time.

However, the IVSC Standards Review Board is in the process of a setting up a working group comprising of IVSC Board members and external specialist to explore this issue further.

Furthermore, the IVS Agenda Consultation, which will go into consultation in 2024 will include Artificial Intelligence as a key topic. Therefore, this will provide respondents with the opportunity to discuss the current role of artificial intelligence within valuation and whether there is a need for more specific standards on this within IVS.

EBVM: What are new projects & topics IVSC is working on which are not yet covered in the actual proposed changes?

Alexander: The IVSC Business Valuation Board will be continuing to publish perspectives papers on internally generated intangible with the next perspective paper on this series on evaluating technology has been published in June 2023 and further perspective papers on that are planned for later this year.

In addition, future perspective papers are planned to be published on a variety of topics including ESG and the nature of valuation risk.

Finally, the IVSC Standards Review Board and Asset Boards will be publishing the Agenda Consultation in 2024 in order to solicit feedback about:

1. The valuation topics that the IVSC should consider as part of its current agenda, and
2. Additional topics that stakeholders feel should be prioritised or added to IVSC's agenda.

The Boards are still in the early stages of considering topics to be included within the agenda consultation but from provisional discussions the IVS Agenda Consultation is likely to include some if not all of the following topics:

- *AVMs and Artificial Intelligence*
- *Data and Data Handling*
- *Environmental, Social and Governance (ESG)*
- *Valuation Risk.* ◆

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Industry Betas and Multiples

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General

To derive the provided betas and multiples, only companies from the Eurozone have been considered. The included companies have been grouped on an industry level and on a sub-industry level based on the Global Industry Classification Standard (GICS). In each issue of the journal, aggregates for all eleven main industries and one individually selected sub-industry will be shown. Due to the special characteristics of companies operating in the financial industry (high leverage, leverage as part of the operating business, high dependency on the interest rate level, etc.), we only provide levered betas and equity-based multiples for that industry.

All presented values are based on raw data and raw calculations. They have carefully been checked and evaluated but have not been audited nor have individual values been verified. Certain results may be misleading in your setup or specific context. All results should be critically evaluated and interpreted. The data and usage are at your own risk.

Data source

All data has been obtained from the KPMG Valuation Data Source. The data source provides access to cost of capital parameters from more than 150 countries and sectors as well as peer-group-specific data from over 16,500 companies worldwide. The data covers the period from 2012 to the present. The data is updated monthly and is accessible from anywhere around the clock.
See www.kpmg.de/en/valuation-data-source for details.

Eurozone Cost of Capital Parameters as at 31 May 2023

The typified, uniform risk-free rate based on AAA-rated government bonds currently lies at 2.5% for the Eurozone. It is derived from yield curves based on Svensson parameters and results published by the European Central Bank. The overall long-term market return for the Eurozone is estimated at around 8.5%, leading to a market risk premium of 6.0%. Estimations of the market return rely on historical returns, as well as on forward-looking return estimates and risk premiums based on Eurozone companies with current market share prices and earnings forecasts from financial analysts.

Betas

Levered, debt and unlevered betas are calculated over an observation period of a single five-year period (monthly returns) and for five one-year periods (weekly returns).

Raw levered betas are obtained from a standard OLS regression, with stock returns being the dependent variable and stock market index returns (S&P Eurozone BMI Index) being the independent variable. Stock and index returns are total returns, thus including dividends, stock splits, rights issues, etc. (if available). Levered betas below zero and above three are treated as outliers and are excluded.

Unlevered betas have been estimated based on Harris-Pringle, assuming uncertain tax shields and including debt beta:

$$\beta_u = \beta_L \frac{E}{E+D} + \beta_D \frac{D}{E+D},$$

where β_u = unlevered beta, β_d = debt beta, D = net debt, E = market value of equity. Debt betas rely on a company's individual rating on a given date. Annual rating-specific levels of debt betas are extracted from a broad market analysis. Net debt consists of total debt (incl. lease liabilities) + net pensions + minority interest + total preferred equity - total cash - short-term investments. In accordance with the observation period, parameter averages of debt beta, net debt and market equity over the individual periods are applied when unlevering levered betas. Unlevered betas below zero and above two are treated as outliers and are excluded.

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Table 1: Average Levered Industry Betas for five single 1y-periods and one 5y-period

31 May 2023	Average* Levered Betas								
	1-Year, weekly returns							5-Year, monthly returns	
Industries	Comps incl. (Average*)	6/2018 to 5/2019	6/2019 to 5/2020	6/2020 to 5/2021	6/2021 to 5/2022	6/2022 to 5/2023	Average*	Comps incl.	6/2018 to 5/2023
Industrials	*257*	1,15	1,01	1,02	0,85	0,91	0,99	*234*	1,17
Consumer Discretionary	*163*	1,04	1,05	1,01	1,06	1,02	1,03	*147*	1,24
Health Care	*130*	1,06	0,79	0,64	0,72	0,84	0,81	*120*	0,81
Financials	*146*	1,00	0,99	1,06	1,00	0,90	0,99	*132*	1,12
Utilities	*49*	0,56	0,82	0,69	0,56	0,69	0,67	*44*	0,69
Materials	*82*	1,20	1,06	0,96	0,83	1,01	1,01	*77*	1,21
Real Estate	*91*	0,47	0,78	0,77	0,59	0,87	0,69	*78*	0,83
Communication Services	*88*	0,87	0,81	0,80	0,61	0,75	0,77	*80*	0,87
Information Technology	*148*	1,20	0,92	0,73	0,85	0,98	0,94	*139*	1,12
Consumer Staples	*59*	0,64	0,65	0,55	0,72	0,68	0,65	*58*	0,65
Energy	*35*	1,05	0,97	1,22	0,45	0,80	0,90	*34*	1,05

Table 2: Average Industry Leverage for five single 1y-periods and one 5y-period

31 May 2023	Average* Debt-Equity-Ratios								
	1-Year							5-Year	
Industries	Comps incl. (Average*)	6/2018 to 5/2019	6/2019 to 5/2020	6/2020 to 5/2021	6/2021 to 5/2022	6/2022 to 5/2023	Average*	Comps incl.	6/2018 to 5/2023
Industrials	*139*	102.8%	135.8%	60.5%	59.4%	64.0%	84.5%	*167*	61.5%
Consumer Discretionary	*76*	135.1%	156.2%	99.4%	98.3%	217.1%	141.2%	*104*	89.2%
Health Care	*53*	27.9%	22.2%	18.0%	64.2%	756.3%	177.7%	*70*	24.5%
Utilities	*35*	103.8%	91.3%	68.4%	68.7%	72.9%	81.0%	*36*	73.5%
Materials	*53*	86.2%	95.4%	44.9%	46.7%	51.3%	64.9%	*58*	52.2%
Real Estate	*41*	94.9%	136.7%	118.7%	132.7%	346.3%	165.8%	*55*	113.4%
Communication Services	*44*	86.1%	374.1%	340.0%	66.1%	70.4%	187.4%	*55*	65.3%
Information Technology	*69*	26.5%	34.1%	10.0%	16.1%	21.2%	21.6%	*91*	12.0%
Consumer Staples	*40*	271.2%	172.5%	156.0%	241.4%	787.8%	325.8%	*46*	192.5%
Energy	*20*	145.8%	428.7%	489.0%	71.5%	61.1%	239.2%	*23*	89.3%

Table 3: Average Unlevered Industry Betas for five single 1y-periods and one 5y-period

31 May 2023	Average* Unlevered Betas								
	1-Year, weekly returns							5-Year, monthly returns	
Industries	Comps incl. (Average*)	6/2018 to 5/2019	6/2019 to 5/2020	6/2020 to 5/2021	6/2021 to 5/2022	6/2022 to 5/2023	Average*	Comps incl.	6/2018 to 5/2023
Industrials	*139*	0.95	0.88	0.86	0.77	0.80	0.85	*167*	0.97
Consumer Discretionary	*76*	0.85	0.87	0.86	0.86	0.82	0.85	*104*	0.97
Health Care	*53*	0.85	0.74	0.57	0.62	0.74	0.70	*70*	0.74
Utilities	*35*	0.55	0.71	0.56	0.44	0.47	0.55	*36*	0.55
Materials	*53*	0.94	0.81	0.79	0.70	0.87	0.82	*58*	0.93
Real Estate	*41*	0.45	0.69	0.60	0.48	0.53	0.55	*55*	0.63
Communication Services	*44*	0.77	0.72	0.65	0.57	0.66	0.67	*55*	0.75
Information Technology	*69*	1.18	1.04	0.77	0.90	0.94	0.97	*91*	1.06
Consumer Staples	*40*	0.62	0.64	0.52	0.60	0.49	0.58	*46*	0.56
Energy	*20*	0.95	0.96	1.04	0.45	0.74	0.83	*23*	0.91

Source: KPMG Valuation Data Source, see www.kpmg.de/en/valuation-data-source

*Average = Arithmetic Mean

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Table 4: Average Levered Subindustry (Utilities) Betas for five single 1y-periods and one 5y-period

31 May 2023	Average* Levered Betas								
	1-Year, weekly returns							5-Year, monthly returns	
Subindustry: Utilities	Comps incl. (Average*)	6/2018 to 5/2019	6/2019 to 5/2020	6/2020 to 5/2021	6/2021 to 5/2022	6/2022 to 5/2023	Average*	Comps incl.	6/2018 to 5/2023
Electric Utilities	*14*	0.52	0.89	0.73	0.64	0.62	0.68	*14*	0.69
Gas Utilities	*6*	0.50	0.75	0.71	0.36	0.68	0.60	*6*	0.78
Independent Power and Renewable Electricity Producers	*18*	0.70	0.81	0.70	0.55	0.70	0.69	*14*	0.61
Multi-Utilities	*10*	0.46	0.82	0.65	0.60	0.89	0.68	*9*	0.78
Water Utilities	*1*	0.72	0.51	0.33	0.65	0.16	0.47	*1*	0.47

Table 5: Average Subindustry (Utilities) Leverage for five single 1y-periods and one 5y-period

31 May 2023	Average* Debt-Equity-Ratios								
	1-Year							5-Year	
Subindustry: Utilities	Comps incl. (Average*)	6/2018 to 5/2019	6/2019 to 5/2020	6/2020 to 5/2021	6/2021 to 5/2022	6/2022 to 5/2023	Average*	Comps incl.	6/2018 to 5/2023
Electric Utilities	*10*	135.2%	115.6%	77.0%	65.4%	71.2%	0.93	*14*	78.7%
Gas Utilities	*5*	58.6%	71.4%	70.5%	74.3%	85.0%	0.72	*6*	69.6%
Independent Power and Renewable Electricity Producers	*11*	95.2%	64.4%	44.4%	51.2%	50.0%	0.61	*6*	52.5%
Multi-Utilities	*7*	112.1%	131.0%	110.1%	115.2%	124.4%	1.19	*9*	112.5%
Water Utilities	*1*	-42.2%	-41.7%	-35.1%	-18.0%	-24.0%	-0.32	*1*	-32.7%

Table 6: Average Unlevered Subindustry (Utilities) Betas for five single 1y-periods and one 5y-period

31 May 2023	Average* Unlevered Betas								
	1-Year, weekly returns							5-Year, monthly returns	
Subindustry: Utilities	Comps incl. (Average*)	6/2018 to 5/2019	6/2019 to 5/2020	6/2020 to 5/2021	6/2021 to 5/2022	6/2022 to 5/2023	Average*	Comps incl.	6/2018 to 5/2023
Electric Utilities	*10*	0.47	0.67	0.53	0.49	0.46	0.52	*14*	0.54
Gas Utilities	*5*	0.39	0.64	0.54	0.23	0.46	0.45	*6*	0.60
Independent Power and Renewable Electricity Producers	*11*	0.70	0.79	0.65	0.41	0.50	0.61	*6*	0.55
Multi-Utilities	*7*	0.39	0.62	0.47	0.46	0.51	0.49	*9*	0.52
Water Utilities	*1*	1.24	0.88	0.51	1.05	0.21	0.78	*1*	0.71

Source: KPMG Valuation Data Source, see www.kpmg.de/en/valuation-data-source

*Average = Arithmetic Mean

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Multiples

Multiples are computed based on actuals (based on the annual report) and forecasts (based on consensus estimates by analyst) for the trailing year and the forward +1 year. Trading multiples for Sales, EBITDA and EBIT are each derived by dividing a companies' enterprise value (market capitalization plus net debt) by its sales, EBITDA or EBIT. Earnings multiples are derived by dividing a companies' market capitalization by earnings (net incoe). The market-to-book ratio is derived by dividing a companies' market value of equity by its book value of equity. Multiples below zero and above 500 are treated as outliers and are excluded. ◆

Table 7: Average Industry Multiples

31 May 2023	Sales			EBITDA			EBIT			Earnings			Market to Book-Ratio		
Industries	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.
Industrials	1.5	1.4	*235*	9.0	8.1	*210*	14.9	12.4	*226*	16.0	15.1	*219*	2.3	2.2	*219*
Consumer Discretionary	1.5	1.3	*147*	9.9	7.8	*126*	15.8	13.7	*141*	17.4	12.6	*132*	2.5	2.5	*136*
Health Care	7.7	12.8	*111*	19.1	11.0	*78*	17.8	19.5	*83*	24.5	19.5	*75*	3.1	2.8	*85*
Financials	n/m	n/m	*n/a*	n/m	n/m	*n/a*	n/m	n/m	*n/a*	n/m	n/m	*114*	n/m	n/m	*109*
Utilities	4.2	3.8	*46*	9.6	8.8	*45*	17.3	15.2	*46*	18.9	19.8	*47*	2.1	1.9	*44*
Materials	1.8	1.3	*74*	6.8	6.6	*67*	13.9	10.1	*73*	13.4	11.2	*72*	1.5	1.3	*68*
Real Estate	13.1	12.1	*66*	20.5	17.1	*62*	30.3	19.8	*65*	15.0	11.5	*59*	0.6	0.6	*54*
Communication Services	2.3	2.1	*78*	7.4	8.3	*69*	22.2	14.8	*76*	16.8	19.0	*72*	2.8	2.2	*64*
Information Technology	2.4	1.9	*139*	20.6	9.9	*113*	25.2	15.6	*123*	23.5	20.3	*116*	4.0	3.5	*117*
Consumer Staples	1.9	1.4	*57*	17.1	10.0	*41*	15.3	13.5	*57*	16.3	14.9	*55*	2.2	2.1	*49*
Energy	1.6	1.3	*34*	7.6	4.5	*30*	8.0	6.8	*33*	12.0	8.0	*33*	1.6	1.4	*30*

Table 8: Average Subindustry (Utilities) Multiples

31 May 2023	Sales			EBITDA			EBIT			Earnings			Market to Book		
Subindustry: Utilities	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.	Trailing	Fwd. +1	Comps incl.
Electric Utilities	3.2	3.2	*14*	8.3	8.0	*13*	13.8	13.3	*14*	14.5	14.2	*14*	1.8	1.7	*13*
Gas Utilities	5.2	5.0	*6*	9.9	9.3	*6*	18.7	16.2	*6*	15.1	14.0	*6*	1.7	1.7	*6*
Independent Power and Renewable Electricity Producers	5.8	5.2	*17*	11.7	10.7	*17*	21.4	17.9	*17*	27.2	30.2	*18*	2.9	2.4	*17*
Multi-Utilities	1.4	1.3	*9*	6.5	6.1	*9*	12.9	12.2	*9*	11.7	11.7	*9*	1.2	1.2	*8*
Water Utilities	n/a	n/a	*0*	n/a	n/a	*0*	n/a	n/a	*0*	n/a	n/a	*0*	n/a	n/a	*0*

Source: KPMG Valuation Data Source, see www.kpmg.de/en/valuation-data-source

*Average = Arithmetic Mean

Transaction Multiples

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The computations of the transaction multiples are based on the transaction and company data collected from various M&A databases, with the data being driven to consistency.

We publish transaction multiples for Europe and resulting regression parameters (including transactions of the period *1 January 2020 until 31 December 2022*) for the following multiples:

- Deal Enterprise Value/Sales
- Deal Enterprise Value/EBITDA
- Deal Enterprise Value/EBIT
- Deal Enterprise Value/Invested Capital

In the previous issue we provided multiples for Europe in total. The multiples **in this issue** provide a regional breakdown into:

- **Central and Western Europe** and
- **Southern Europe**

In the following issue we will continue the regional breakdown into Scandinavia and Britain and Eastern Europe.

When using the data (multiples and regression), please consider the following:

- Sectors and resulting sector multiples are formed according to the *NACE Rev. 2 industry classification system.*
- The multiples indicate the Deal Enterprise Value *(DEPV = Market value of total capital corrected)* for a private firm. They are scaled to the levels of value Control Value, Pure Play Value and Domestic Value. Additionally, the multiples *do not include any identifiable Synergistic Values*. When applying the multiples to other levels of value without adjusting the value driver (reference value), respective *Valuation Adjustments* (Minority Discount for Minority Values, Conglomerate Discount for Conglomerates, Regional Premiums for Cross-Border transactions by international acquirors and Strategic Premium for Synergistic acquisitions) must be applied.
- The multiples are computed using transaction data collected from the previous three years. Therefore, the available multiples include transactions of the period *1 January 2020 until 31 December 2022*, with the transactions of the *latest six months given double weight*.
- The reliability of the recorded transaction data and the resulting multiples was analyzed according to the fraction of the transacted share, low and high values of the value driver as well as up-side and down-side percentiles of the observations on multiples; recognized outliers were eliminated.
- Trailing multiples are computed employing the value driver available closest to date of the transaction. Forward multiples are computed using mean and/or median estimates for the forthcoming three to six years after the transaction (not available for Invested Capital).
- The EBITDA multiples and the EBIT multiples are based on companies with only a positive EBITDA or EBIT at date of the transaction.
- The regression assumes a linear relationship between the value driver and the Deal Enterprise Value. Furthermore, it is assumed that the observed Deal Enterprise Values as well as the respective value drivers show no trend over time, making them ready for a cross-section analysis. The error terms are assumed to be normally distributed, having constant variances (homoskedasticity), being independent (no autocorrelation) and showing an expected value of zero.
- The range of the multiples (confidence interval) applies a 95% confidence level, assuming the observed multiples to be normally distributed (after elimination of outliers).
- Sectors with less than 20 observations were ignored.

- The various regions are compounded as follows:
  - **Central and Western Europe:** Andorra, Austria, Belgium, France, Germany, Liechtenstein, Luxembourg, Monaco, The Netherlands, Switzerland
  - **Southern Europe:** Croatia, Cyprus, Gibraltar, Greece, Italy, Malta, Portugal, San Marino, Slovenia, Spain, Turkey
  - **Scandinavia:** Denmark, Finland, Iceland, Norway, Sweden
  - **Britain:** Ireland, United Kingdom
  - **Eastern Europe:** Albania, Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Bulgaria, Czech Republic, Estonia, Georgia, Hungary, Kazakhstan, Kosovo, Latvia, Lithuania, Moldova, Montenegro, North Makedonia, Poland, Romania, Russia, Serbia, Slovakia, Ukraine.

The data is evaluated carefully; however, the author denies liability for the accuracy of all computations.

Notes for application:

n indicates the number of observations (sample size) included in both, the computation of the multiples and the regression. $\bar{x}_a$ indicates the arithmetic mean, $\bar{x}_h$ indicates the harmonic mean

$$\bar{x}_h = \frac{n}{\sum_{i=1}^{n} \frac{n}{x_i}}$$

and $\bar{x}_t$ indicates the truncated mean (10% level = 10 % of the observations sorted in ascending order being eliminated up-side and down-side)

$$\bar{x}_t = \frac{\sum_2^{n-1} x_i}{n-2}$$

The first quartile Q_1 indicates the boundary of the lowest 25%, the third quartile Q_3 indicates the boundary of the highest 25% of the observed multiples. Using this information, the actually employed multiple may be related to the group of the 25% lowest (highest) multiples observed. Q_2 indicates the median of the observed multiples. The confidence interval reports the range (lower confidence limit to upper confidence limit) of the multiples applying a 95% confidence level. Assuming the multiples observed to be normally distributed, this indicates all multiples lying within these limits. To evaluate the assumption of normally distributed multiple observations, the results of the Jarque-Bera Test for Normality are reported in brackets

$$JB = n\left[\frac{(skewness)^2}{6} + \frac{(kurtosis-3)^2}{24}\right]$$

Values above the reported 5% significance points reject the null hypothesis of normality, indicating the confidence interval to be less reliable:

n	5%	n	5%	n	5%	n	5%
100	4,29	200	4,43	400	4,74	800	5,46
150	4,39	300	4,6	500	4,82	∞	5,99

The skewness **sk** indicates the symmetry of the distribution of multiple observations. A negative skewness indicates the distribution to be skewed to the left, whereas a positive skewness indicates the distribution to be skewed to the right (a skewness of zero indicates the distribution to be symmetric). The coefficient of variation **cv** indicates the dispersion of the observed multiples adjusting for the scale of units in the multiples, expressed by the standard deviation as a percentage of the mean. It allows for a comparison of the dispersion of the multiples across sectors. A lower (higher) coefficient of variation indicates a lower (higher) dispersion of the observed multiples and, similarly, a higher (lower) reliability of the sector multiples.

The (linear) regression equation allows for computing the Deal Enterprise Value of a private firm directly from the observed transactions (without using a multiple). Disregarding the error term, it consists of a slope expressed in terms of the value driver employed and a constant (intercept)

$$\hat{y}=\text{DEPV}=\text{slope x value driver}+\text{constant}(+\text{error term})$$

The reliability of the OLS regression equation (goodness of fit) is indicated by the adjusted coefficient of determination

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-p}$$

(with **p** indicating the number of explaining variables + 1 = 1 + 1 = 2; being sensitive to the number of observations), indicating the variability of the observed multiples that is explained by the regression equation. Unlike the (unadjusted) coefficient of determination, the adjusted coefficient of determination is not limited to the range between zero and one. A higher (lower) coefficient indicates a better (poorer) regression. The standard error of the regression equation similarly indicates the goodness of fit of the regression equation, indicating the degree of similarity between the regression residuals (error terms) and the “true” residuals. A lower (higher) standard error indicates a better (poorer) regression. ◆

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Central and Western Europe - Trailing & Forward DEPV/Sales (operating), 1 January 2020 until 31 December 2022

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
C19 - C23	Manufacture of coke, chemicals, rubber, refined petroleum/chemical/pharmaceutical/plastic/mineral products
C24 - C25	Manufacture of basic metals, fabricated metal products
C26 - C27	Manufacture of computers, electronic/optical products, electrical equipment
C28 - C30, C33	Manufacture of machinery, motor vehicles, other transport equipment; repair/installation
D35	Electricity, gas, steam and air conditioning supply
E36 - E39	Water supply, sewerage, waste management, remediation activities
F41 - F43	Construction - Buildings, civil engineering, specialized construction activities
G45 - G47	Wholesale/Retail trade, repair of motor vehicles and motorcycles
H49 - H53	Transportation and storage - Land/pipelines, water, air; warehousing, postal/courier activities
J58 - J60, C18	Publishing activities, programme production, music publishing, broadcasting, printing
J61 - J63	Telecommunications, computer programming/consultancy, information service activities
K64 - K66	Financial and insurance activities
L68	Real estate activities
M69, M70, M73, N77 - N82	Legal/accounting activities, consultancy, advertising/market research, rental/employment/security activities, travel agency
M71, M72, M74, M75	Architectural/engineering/other professional activities, technical testing, scientific R&D, veterinary activities

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
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n	Trailing DEPV/Sales (operating) Multiples									Trailing Sales (operating) Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
445	1.70	1.06	1.75	1.10	1.81	2.31	[1,59 ; 1,81] (56,1)	-0.47	0.42	$\hat{y}$ = 1,902 x Sales + 652.635	0.86	3,689,311
1,020	2.02	1.72	2.05	1.52	2.06	2.56	[1,96 ; 2,08] (119,5)	-0.26	0.32	$\hat{y}$ = 2,377 x Sales - 355.258	0.67	706,841
392	1.71	0.69	1.75	0.64	2.09	2.65	[1,47 ; 1,95] (65,7)	-0.28	0.60	$\hat{y}$ = 2,437 x Sales - 264.936	0.83	854,915
2,174	1.90	0.45	1.95	1.38	2.01	2.45	[1,85 ; 1,95] (218,2)	-0.52	0.38	$\hat{y}$ = 2,214 x Sales - 830.241	0.93	3,755,802
1,057	0.95	0.06	0.87	0.61	0.68	1.23	[0,90 ; 0,99] (86,1)	1.23	0.62	$\hat{y}$ = 0,285 x Sales + 741.281	0.91	900,941
4,154	1.00	0.71	0.87	0.53	0.66	1.22	[0,96 ; 1,03] (400,2)	1.43	0.68	$\hat{y}$ = 1,307 x Sales - 2.533.841	0.61	4,138,121
2,115	1.29	0.11	1.26	0.58	1.20	2.10	[1,22 ; 1,36] (305,9)	0.30	0.64	$\hat{y}$ = 0,497 x Sales + 4.417.908	0.43	16,586,476
70	1.37	0.27	1.38	0.72	1.85	1.94	[0,99 ; 1,75] (11,5)	-0.19	0.61	$\hat{y}$ = 1,965 x Sales - 578.841	0.98	2,549,936
81	0.78	0.35	0.70	0.22	0.29	1.26	[0,49 ; 1,06] (7,2)	1.12	0.97	$\hat{y}$ = 0,192 x Sales + 1.423.533	0.07	3,139,561
397	0.52	0.10	0.39	0.08	0.19	0.81	[0,43 ; 0,62] (43,4)	1.84	1.24	$\hat{y}$ = -0,022 x Sales + 910.360	-0.01	976,241
2,115	1.05	0.41	0.97	0.44	0.70	1.67	[0,99 ; 1,12] (263,5)	0.85	0.78	$\hat{y}$ = 0,562 x Sales + 1.326.124	0.86	2,369,224
2,404	1.00	0.68	0.90	0.58	0.69	1.55	[0,96 ; 1,04] (261,2)	1.25	0.66	$\hat{y}$ = 0,520 x Sales + 2.625.533	0.25	4,083,189
1,261	1.76	1.12	1.81	1.21	1.62	2.61	[1,68 ; 1,84] (157,5)	-0.26	0.44	$\hat{y}$ = 1,774 x Sales + 1.123.612	0.93	3,672,503
2,372	1.82	0.90	1.89	1.21	1.85	2.67	[1,75 ; 1,89] (314,9)	-0.46	0.46	$\hat{y}$ = 1,728 x Sales + 1.328.673	0.93	2,729,283
113	1.37	0.39	1.37	0.68	1.11	2.02	[1,03 ; 1,72] (15,7)	0.27	0.65	$\hat{y}$ = 0,803 x Sales + 113.504	1.00	702,639
43	1.49	0.19	1.49	0.18	1.62	2.49	[0,49 ; 2,49] (8,7)	-0.01	0.81	$\hat{y}$ = 3,463 x Sales - 107.789	0.71	84,966
725	0.69	0.23	0.53	0.30	0.34	0.67	[0,59 ; 0,79] (76,7)	1.76	1.13	$\hat{y}$ = 0,325 x Sales - 8.150	0.94	890,870
1,717	1.54	0.56	1.55	0.80	1.48	2.39	[1,46 ; 1,62] (248,1)	-0.04	0.55	$\hat{y}$ = 1,041 x Sales + 963.511	0.31	2,608,872

n	Forward DEPV/Sales (operating) Multiples									Forward Sales (operating) Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
687	1.97	1.09	2.03	1.64	2.11	2.34	[1,90 ; 2,04] (39,7)	-1.04	0.32	$\hat{y}$ = 2,026 x Sales - 813.968	0.95	11,045,101
2,447	0.98	0.72	0.84	0.60	0.73	0.91	[0,94 ; 1,02] (256,9)	1.77	0.69	$\hat{y}$ = 0,916 x Sales + 9.590	0.74	5,628,128
1,465	1.36	0.94	1.30	0.95	1.12	1.48	[1,29 ; 1,42] (157,3)	0.82	0.55	$\hat{y}$ = 0,531 x Sales + 2.304.177	0.57	3,826,998
10,364	0.90	0.40	0.81	0.54	0.72	1.10	[0,88 ; 0,91] (893,5)	1.66	0.59	$\hat{y}$ = 0,506 x Sales + 3.978.261	0.58	13,745,183
2,415	0.39	0.26	0.34	0.24	0.30	0.42	[0,38 ; 0,40] (318,3)	1.97	0.71	$\hat{y}$ = 0,220 x Sales + 840.491	0.61	2,666,767
7,589	0.91	0.64	0.80	0.47	0.67	1.15	[0,89 ; 0,93] (653,6)	1.51	0.68	$\hat{y}$ = 0,741 x Sales - 252.378	0.75	4,621,021
9,838	0.51	0.33	0.43	0.26	0.37	0.55	[0,50 ; 0,52] (1.306,9)	2.00	0.79	$\hat{y}$ = 0,309 x Sales + 1.697.474	0.60	8,302,607
2,281	0.51	0.20	0.38	0.27	0.37	0.46	[0,48 ; 0,54] (1.067,6)	2.93	1.09	$\hat{y}$ = 0,067 x Sales + 5.862.076	0.12	6,802,135
1,159	0.36	0.28	0.32	0.22	0.29	0.39	[0,35 ; 0,37] (9.635,9)	4.94	0.75	$\hat{y}$ = 0,334 x Sales + 806.125	0.73	4,217,957
2,785	0.46	0.24	0.35	0.21	0.31	0.45	[0,44 ; 0,48] (2.820,2)	3.41	1.13	$\hat{y}$ = 0,540 x Sales - 938.930	0.67	6,133,674
6,483	0.58	0.24	0.45	0.20	0.35	0.68	[0,56 ; 0,61] (928,3)	2.08	1.05	$\hat{y}$ = 0,290 x Sales + 876.803	0.27	8,487,807
4,959	0.50	0.19	0.35	0.17	0.25	0.40	[0,48 ; 0,53] (770,6)	2.19	1.25	$\hat{y}$ = 0,142 x Sales + 2.927.660	0.25	3,701,439
4,031	1.31	0.73	1.26	0.55	1.11	1.98	[1,26 ; 1,36] (585,4)	0.34	0.64	$\hat{y}$ = 0,996 x Sales + 840.414	0.54	10,793,855
11,212	1.28	0.65	1.23	0.62	1.09	2.00	[1,25 ; 1,31] (1.570,3)	0.40	0.62	$\hat{y}$ = 0,700 x Sales + 2.714.884	0.34	8,498,647
558	1.65	0.75	1.67	0.69	1.76	2.38	[1,49 ; 1,81] (81,2)	-0.32	0.55	$\hat{y}$ = 0,247 x Sales + 3.448.402	0.46	10,689,010
86	1.53	0.60	1.54	0.61	1.99	2.27	[1,11 ; 1,95] (14,9)	-0.31	0.61	$\hat{y}$ = 0,499 x Sales + 207.635	0.58	788,348
3,521	0.77	0.33	0.66	0.25	0.48	1.20	[0,73 ; 0,80] (388,4)	1.09	0.90	$\hat{y}$ = 0,284 x Sales + 2.521.554	0.27	5,606,364
4,358	1.06	0.44	0.97	0.31	0.81	1.60	[1,02 ; 1,11] (538,5)	0.74	0.79	$\hat{y}$ = 0,457 x Sales + 865.496	0.20	12,040,072

Central and Western Europe - Trailing & Forward DEPV/EBITDA, 1 January 2020 until 31 December 2022

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
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E36 - E39	Water supply, sewerage, waste management, remediation activities
F41 - F43	Construction - Buildings, civil engineering, specialized construction activities
G45 - G47	Wholesale/Retail trade, repair of motor vehicles and motorcycles
H49 - H53	Transportation and storage - Land/pipelines, water, air; warehousing, postal/courier activities
J58 - J60, C18	Publishing activities, programme production, music publishing, broadcasting, printing
J61 - J63	Telecommunications, computer programming/consultancy, information service activities
K64 - K66	Financial and insurance activities
L68	Real estate activities
M69, M70, M73, N77 - N82	Legal/accounting activities, consultancy, advertising/market research, rental/employment/security activities, travel agency
M71, M72, M74, M75	Architectural/engineering/other professional activities, technical testing, scientific R&D, veterinary activities

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
C19 - C23	Manufacture of coke, chemicals, rubber, refined petroleum/chemical/pharmaceutical/plastic/mineral products
C24 - C25	Manufacture of basic metals, fabricated metal products
C26 - C27	Manufacture of computers, electronic/optical products, electrical equipment
C28 - C30, C33	Manufacture of machinery, motor vehicles, other transport equipment; repair/installation
D35	Electricity, gas, steam and air conditioning supply
E36 - E39	Water supply, sewerage, waste management, remediation activities
F41 - F43	Construction - Buildings, civil engineering, specialized construction activities
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n	Trailing DEPV/EBITDA Multiples									Trailing EBITDA Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
440	11.63	10.48	11.68	9.72	11.61	13.74	[9,52 ; 13,74] (30,6)	-0.17	0.27	$\hat{y}$ = 12,858 x EBITDA - 1.550.649	0.93	11,715,171
392	7.63	7.00	7.79	7.28	7.77	8.50	[7,15 ; 8,11] (41,3)	-1.56	0.19	$\hat{y}$ = 7,309 x EBITDA + 374.238	0.71	1,091,573
521	7.84	6.85	7.69	6.97	7.75	8.54	[6,49 ; 9,18] (17,3)	1.01	0.33	$\hat{y}$ = 7,515 x EBITDA + 199.558	0.90	1,011,547
1,675	5.68	1.43	5.15	0.76	2.25	10.69	[2,45 ; 8,91] (253,6)	0.46	0.95	$\hat{y}$ = -0,670 x EBITDA + 9.371.234	0.05	6,501,626
1,513	8.42	1.50	7.95	6.65	7.16	9.74	[7,06 ; 9,78] (106,3)	1.23	0.41	$\hat{y}$ = 3,646 x EBITDA + 2.416.739	0.65	2,445,617
4,133	7.58	6.08	6.93	4.80	5.65	9.85	[6,47 ; 8,68] (377,0)	1.27	0.52	$\hat{y}$ = 13,519 x EBITDA - 3.998.615	0.83	4,190,553
2,710	9.11	2.25	8.81	6.56	7.35	11.80	[7,75 ; 10,47] (249,2)	0.61	0.43	$\hat{y}$ = 3,316 x EBITDA + 3.960.316	0.32	8,637,683
54	11.28	8.32	11.36	8.06	12.46	13.49	[-2,63 ; 25,18] (5,3)	-0.30	0.42	$\hat{y}$ = 10,902 x EBITDA - 12.097	0.98	495,744
16	-	-	-	-	-	-	-	-	-	-	-	-
360	4.74	2.83	4.27	2.54	2.79	6.55	[1,86 ; 7,61] (23,1)	1.41	0.73	$\hat{y}$ = 6,973 x EBITDA - 487.138	0.52	705,205
1,868	11.05	8.16	11.13	7.51	12.14	14.05	[9,01 ; 13,08] (217,5)	-0.18	0.40	$\hat{y}$ = 9,635 x EBITDA + 217.682	0.72	6,653,093
1,487	11.01	8.98	11.26	8.77	12.24	13.67	[9,37 ; 12,64] (153,4)	-0.56	0.34	$\hat{y}$ = 8,893 x EBITDA + 610.335	0.82	1,254,986
1,127	7.25	3.62	6.61	4.84	5.71	9.02	[4,45 ; 10,05] (95,7)	1.24	0.63	$\hat{y}$ = 4,966 x EBITDA + 1.362.173	0.71	8,222,462
1,943	7.13	3.38	6.56	5.01	5.72	9.21	[5,25 ; 9,00] (138,0)	1.32	0.60	$\hat{y}$ = 4,777 x EBITDA + 1.173.633	0.82	4,774,401
134	7.82	3.68	7.90	2.62	9.63	11.61	[-0,38 ; 16,03] (22,2)	-0.30	0.58	$\hat{y}$ = 4,486 x EBITDA + 891.362	0.22	1,851,511
204	11.74	7.50	11.80	7.20	10.81	17.39	[2,71 ; 20,77] (28,5)	0.09	0.45	$\hat{y}$ = 8,276 x EBITDA + 122.680	0.62	351,560
1,020	6.43	2.81	6.03	5.24	5.99	7.08	[4,72 ; 8,13] (134,0)	1.82	0.54	$\hat{y}$ = 5,799 x EBITDA + 241.791	0.85	1,131,071
1,841	9.74	7.47	9.57	6.65	8.89	13.00	[7,90 ; 11,57] (194,2)	0.37	0.43	$\hat{y}$ = 6,343 x EBITDA + 1.431.446	0.70	1,769,059

n	Forward DEPV/EBITDA Multiples									Forward EBITDA Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
757	9.18	7.71	9.28	6.92	9.26	11.85	[7,64 ; 10,72] (79,9)	-0.23	0.33	$\hat{y}$ = 6,425 x EBITDA + 4.628.740	0.87	16,463,357
2,522	5.44	4.00	4.93	3.31	5.10	6.46	[4,52 ; 6,37] (191,4)	1.54	0.59	$\hat{y}$ = 10,550 x EBITDA - 5.769.065	0.75	5,356,409
1,594	6.07	5.28	5.64	5.07	5.49	6.44	[5,24 ; 6,90] (506,1)	2.42	0.45	$\hat{y}$ = 3,497 x EBITDA + 1.554.361	0.66	3,403,225
10,385	4.93	2.07	4.37	3.10	4.38	5.57	[4,54 ; 5,32] (1.698,6)	2.09	0.60	$\hat{y}$ = 2,497 x EBITDA + 4.469.517	0.64	12,718,787
2,437	3.55	2.68	3.37	1.92	3.55	4.41	[3,23 ; 3,86] (394,7)	1.68	0.52	$\hat{y}$ = 2,963 x EBITDA + 673.946	0.67	2,456,831
7,723	5.35	3.97	4.80	3.28	4.45	6.08	[4,80 ; 5,89] (909,5)	1.88	0.61	$\hat{y}$ = 7,785 x EBITDA - 2.513.750	0.66	11,596,435
9,532	4.70	3.34	4.19	3.00	4.10	5.20	[4,28 ; 5,12] (2.899,9)	2.39	0.64	$\hat{y}$ = 2,527 x EBITDA + 3.384.792	0.40	15,854,586
2,367	6.14	4.20	5.37	3.52	4.96	7.23	[4,53 ; 7,75] (212,6)	1.63	0.68	$\hat{y}$ = 2,520 x EBITDA + 2.252.460	0.79	3,298,992
1,159	3.07	2.61	2.96	1.99	3.12	3.69	[2,85 ; 3,30] (523,4)	2.00	0.42	$\hat{y}$ = 2,290 x EBITDA + 714.705	0.76	3,970,430
3,891	6.35	3.98	5.83	3.51	4.63	8.82	[5,07 ; 7,64] (388,7)	0.99	0.66	$\hat{y}$ = 3,296 x EBITDA + 2.330.201	0.70	5,116,001
6,194	5.34	3.00	4.73	2.39	4.33	6.99	[4,43 ; 6,25] (466,4)	1.34	0.74	$\hat{y}$ = 6,851 x EBITDA - 3.788.937	0.56	14,636,795
5,276	3.60	1.68	3.14	1.60	2.28	5.02	[3,01 ; 4,19] (813,0)	1.94	0.85	$\hat{y}$ = 1,134 x EBITDA + 3.148.748	0.26	3,574,706
4,213	6.95	5.23	6.58	4.02	5.83	9.38	[6,03 ; 7,87] (328,6)	0.87	0.52	$\hat{y}$ = 5,811 x EBITDA + 273.629	0.80	7,909,383
8,169	6.35	4.19	5.71	3.35	5.46	8.02	[5,52 ; 7,18] (601,8)	1.47	0.64	$\hat{y}$ = 4,818 x EBITDA + 744.464	0.69	7,323,495
1,739	7.90	5.98	7.48	4.61	6.92	10.97	[6,04 ; 9,76] (180,0)	0.80	0.52	$\hat{y}$ = 1,879 x EBITDA + 1.868.587	0.64	5,050,127
1,744	10.97	9.52	10.96	8.39	11.01	13.73	[9,65 ; 12,30] (164,9)	0.03	0.32	$\hat{y}$ = 10,475 x EBITDA + 113.659	0.84	3,079,291
3,671	5.08	3.46	4.57	2.49	4.14	6.53	[4,24 ; 5,91] (251,3)	1.42	0.66	$\hat{y}$ = 2,688 x EBITDA + 2.220.729	0.51	4,627,685
4,353	5.54	3.36	4.85	2.83	4.45	6.72	[4,44 ; 6,64] (351,0)	1.45	0.72	$\hat{y}$ = 5,542 x EBITDA - 1.811.220	0.72	7,155,899

Central and Western Europe - Trailing & Forward DEPV/EBIT, 1 January 2020 until 31 December 2022

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C28 - C30, C33	Manufacture of machinery, motor vehicles, other transport equipment; repair/installation
D35	Electricity, gas, steam and air conditioning supply
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n	Trailing DEPV/EBIT Multiples									Trailing EBIT Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
440	16.61	13.95	16.61	12.57	15.88	20.23	[9,90 ; 23,33] (36,5)	-0.03	0.34	ŷ = 15,203 x EBIT + 1.890.693	0.93	11,212,506
27	14.88	4.60	14.88	4.81	16.95	23.39	[-78,22 ; 107,98] (6,3)	-0.12	0.69	ŷ = 28,564 x EBIT - 89.288	0.99	109,406
553	23.75	16.82	25.17	24.28	26.85	27.58	[13,35 ; 34,15] (54,3)	-1.63	0.31	ŷ = 24,764 x EBIT + 53.381	0.82	426,410
864	15.61	11.63	15.55	12.20	15.22	19.65	[9,53 ; 21,68] (77,3)	0.13	0.40	ŷ = 13,704 x EBIT + 1.674.020	0.76	4,082,957
1,487	10.43	9.36	10.10	7.25	10.65	11.53	[9,11 ; 11,75] (57,6)	0.91	0.32	ŷ = 6,840 x EBIT + 1.113.517	0.82	1,738,715
4,208	11.81	9.70	10.97	7.89	9.49	14.59	[9,75 ; 13,87] (341,3)	1.23	0.46	ŷ = 23,095 x EBIT - 4.474.273	0.95	5,351,046
2,539	11.31	8.27	10.61	7.09	10.66	14.00	[8,69 ; 13,93] (157,9)	1.20	0.48	ŷ = 17,965 x EBIT - 5.711.892	0.72	15,038,633
43	16.79	14.92	16.79	13.75	15.93	20.85	[-3,62 ; 37,20] (5,4)	0.08	0.32	ŷ = 20,223 x EBIT - 133.729	0.99	317,046
38	6.12	4.62	6.12	3.73	4.54	6.13	[-6,18 ; 18,41] (4,6)	1.67	0.67	ŷ = 2,582 x EBIT + 29.138	0.96	20,516
284	8.89	6.08	7.56	5.65	5.95	9.64	[-3,40 ; 21,18] (27,1)	1.69	0.76	ŷ = 25,583 x EBIT - 1.331.856	0.96	879,964
1,556	15.77	9.96	15.81	12.19	14.43	21.86	[9,67 ; 21,87] (174,3)	0.02	0.46	ŷ = 23,275 x EBIT - 1.688.853	0.96	7,446,612
1,304	16.31	13.54	16.14	12.91	15.00	20.69	[11,87 ; 20,75] (110,5)	0.31	0.36	ŷ = 18,979 x EBIT - 532.493	0.64	1,811,372
977	13.36	5.61	13.46	10.32	14.27	15.55	[9,51 ; 17,20] (9,9)	0.01	0.39	ŷ = 12,836 x EBIT + 533.859	0.87	5,540,957
2,147	15.06	5.63	15.10	10.82	14.61	19.07	[10,58 ; 19,53] (177,0)	0.08	0.45	ŷ = 12,412 x EBIT + 1.150.017	0.88	3,783,793
161	10.74	4.28	10.29	3.62	12.46	14.45	[-7,18 ; 28,67] (18,2)	0.22	0.66	ŷ = 4,438 x EBIT + 917.566	0.23	1,778,177
215	15.29	9.40	14.93	9.37	15.09	19.70	[0,42 ; 30,17] (20,8)	0.32	0.45	ŷ = 25,409 x EBIT - 474.227	0.98	719,892
1,030	8.47	3.77	7.92	5.81	7.99	9.45	[5,62 ; 11,32] (75,8)	1.51	0.53	ŷ = 7,941 x EBIT + 40.570	0.73	1,506,831
1,315	12.18	9.38	11.27	7.02	10.41	15.35	[6,94 ; 17,42] (116,5)	1.08	0.53	ŷ = 6,185 x EBIT + 1.595.383	0.77	1,627,025

n	Forward DEPV/EBIT Multiples									Forward EBIT Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
757	12.89	10.60	12.65	8.49	12.09	17.66	[8,02 ; 17,77] (90,2)	0.44	0.42	ŷ = 8,366 x EBIT + 5.025.357	0.88	15,632,389
2,501	10.69	7.94	10.00	6.52	8.74	14.35	[7,59 ; 13,78] (186,7)	1.04	0.55	ŷ = 17,574 x EBIT - 3.776.333	0.75	5,278,068
1,567	10.55	8.80	10.10	6.98	8.70	15.19	[8,03 ; 13,08] (181,3)	0.80	0.44	ŷ = 5,958 x EBIT + 1.691.875	0.71	3,099,586
10,074	8.02	3.34	7.57	4.60	7.70	10.49	[7,20 ; 8,83] (608,7)	1.38	0.53	ŷ = 3,334 x EBIT + 5.212.737	0.64	12,816,894
2,437	7.05	4.18	5.91	2.50	6.10	8.44	[4,14 ; 9,97] (279,9)	1.90	0.80	ŷ = 4,026 x EBIT + 1.253.927	0.59	2,768,634
7,412	7.86	5.98	7.04	4.73	5.91	9.67	[6,69 ; 9,03] (614,2)	1.50	0.60	ŷ = 10,332 x EBIT - 2.441.989	0.76	9,914,662
9,280	7.15	5.49	6.84	5.28	6.36	8.49	[6,61 ; 7,70] (3.036,4)	1.95	0.47	ŷ = 3,819 x EBIT + 3.718.765	0.48	14,943,311
2,066	8.16	6.68	7.95	5.71	7.68	10.61	[7,01 ; 9,32] (83,3)	0.76	0.42	ŷ = 4,427 x EBIT + 2.342.664	0.80	3,193,587
1,159	5.31	4.74	5.12	3.94	5.07	6.19	[4,84 ; 5,78] (285,6)	1.80	0.35	ŷ = 4,459 x EBIT + 267.395	0.82	3,473,334
3,902	7.82	5.93	7.31	5.38	6.57	9.31	[6,48 ; 9,16] (702,0)	1.89	0.55	ŷ = 5,280 x EBIT + 1.686.234	0.77	4,550,881
5,839	8.97	5.55	8.31	4.24	6.69	13.51	[6,88 ; 11,06] (531,2)	0.89	0.66	ŷ = 10,362 x EBIT - 2.544.819	0.75	11,333,319
5,276	6.75	4.14	6.13	3.77	5.64	8.62	[5,45 ; 8,05] (571,3)	1.75	0.68	ŷ = 3,404 x EBIT + 2.230.441	0.50	2,942,388
4,176	11.50	9.35	10.94	7.10	10.45	14.72	[9,51 ; 13,48] (301,7)	1.00	0.46	ŷ = 8,547 x EBIT + 1.103.468	0.92	5,006,856
7,836	10.67	8.40	10.17	6.59	9.76	13.34	[9,35 ; 11,98] (508,2)	0.97	0.47	ŷ = 8,209 x EBIT + 892.123	0.87	4,833,171
1,637	10.62	8.76	10.00	7.34	9.75	11.99	[7,92 ; 13,33] (100,8)	1.36	0.46	ŷ = 3,801 x EBIT + 1.489.745	0.78	4,086,021
1,766	11.46	9.86	10.96	8.57	10.09	13.99	[9,20 ; 13,71] (97,9)	1.31	0.40	ŷ = 10,598 x EBIT + 48.859	0.86	2,832,239
3,456	8.15	5.92	7.38	4.23	6.74	10.01	[6,25 ; 10,04] (253,8)	1.39	0.61	ŷ = 5,198 x EBIT + 1.384.878	0.68	3,720,295
4,015	8.95	6.10	8.39	5.08	8.37	11.36	[6,98 ; 10,93] (221,4)	1.16	0.59	ŷ = 7,171 x EBIT - 574.020	0.81	6,055,754

Central and Western Europe - Trailing DEPV/Invested Capital, 1 January 2020 until 31 December 2022

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Southern Europe - Trailing DEPV/Invested Capital, 1 January 2020 until 31 December 2022

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
C19 - C23	Manufacture of coke, chemicals, rubber, refined petroleum/chemical/pharmaceutical/plastic/mineral products
C24 - C25	Manufacture of basic metals, fabricated metal products
C26 - C27	Manufacture of computers, electronic/optical products, electrical equipment
C28 - C30, C33	Manufacture of machinery, motor vehicles, other transport equipment; repair/installation
D35	Electricity, gas, steam and air conditioning supply
E36 - E39	Water supply, sewerage, waste management, remediation activities
F41 - F43	Construction - Buildings, civil engineering, specialized construction activities
G45 - G47	Wholesale/Retail trade, repair of motor vehicles and motorcycles
H49 - H53	Transportation and storage - Land/pipelines, water, air; warehousing, postal/courier activities
J58 - J60, C18	Publishing activities, programme production, music publishing, broadcasting, printing
J61 - J63	Telecommunications, computer programming/consultancy, information service activities
K64 - K66	Financial and insurance activities
L68	Real estate activities
M69, M70, M73, N77 - N82	Legal/accounting activities, consultancy, advertising/market research, rental/employment/security activities, travel agency
M71, M72, M74, M75	Architectural/engineering/other professional activities, technical testing, scientific R&D, veterinary activities

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n	Trailing DEPV/Invested Capital Multiples									Trailing Invested Capital Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
381	0.89	0.73	0.92	0.75	0.92	1.13	[0,87 ; 0,91] (31,0)	-0.84	0.31	ŷ = 0,843 x IC + 1.374.926	0.97	8,407,722
327	0.94	0.77	0.97	0.74	1.03	1.17	[0,92 ; 0,96] (30,7)	-1.01	0.30	ŷ = 1,119 x IC - 112.838	0.79	254,883
483	0.76	0.58	0.77	0.49	0.83	1.05	[0,74 ; 0,78] (69,5)	-0.18	0.42	ŷ = 0,954 x IC - 178.154	0.97	724,329
7,900	0.76	0.34	0.77	0.57	0.78	0.95	[0,76 ; 0,77] (747,7)	-0.16	0.34	ŷ = 0,893 x IC - 1.581.657	0.94	5,570,178
2,989	0.54	0.12	0.51	0.31	0.55	0.65	[0,53 ; 0,54] (240,7)	0.75	0.48	ŷ = 0,231 x IC + 1.150.435	0.82	1,446,278
6,194	0.65	0.54	0.63	0.47	0.57	0.84	[0,64 ; 0,65] (682,8)	0.55	0.40	ŷ = 0,656 x IC - 260.811	0.94	2,169,461
9,269	0.63	0.25	0.61	0.37	0.60	0.84	[0,62 ; 0,63] (1.147,6)	0.24	0.46	ŷ = 0,547 x IC - 583.392	0.77	6,206,022
2,361	0.60	0.39	0.58	0.42	0.54	0.76	[0,59 ; 0,60] (195,7)	0.49	0.41	ŷ = 0,423 x IC + 1.128.067	0.80	3,301,801
1,283	0.41	0.38	0.40	0.33	0.40	0.47	[0,41 ; 0,41] (84,2)	1.24	0.29	ŷ = 0,452 x IC - 447.956	0.89	2,537,651
4,084	0.60	0.41	0.59	0.38	0.57	0.79	[0,60 ; 0,61] (396,9)	0.36	0.47	ŷ = 0,656 x IC + 38.955	0.79	4,649,311
4,433	0.59	0.39	0.57	0.34	0.55	0.80	[0,58 ; 0,59] (525,1)	0.42	0.53	ŷ = 0,434 x IC + 256.558	0.66	3,307,570
4,261	0.44	0.30	0.40	0.23	0.31	0.62	[0,44 ; 0,45] (391,3)	1.04	0.64	ŷ = 0,200 x IC + 2.698.948	0.29	3,796,366
4,084	0.68	0.52	0.67	0.41	0.68	0.88	[0,67 ; 0,69] (517,3)	0.16	0.44	ŷ = 0,562 x IC + 1.005.788	0.84	4,378,494
7,841	0.71	0.51	0.71	0.39	0.69	1.02	[0,70 ; 0,71] (1.149,3)	0.03	0.47	ŷ = 0,539 x IC + 1.139.453	0.76	3,734,903
1,234	0.77	0.46	0.79	0.55	0.78	1.03	[0,76 ; 0,78] (121,7)	-0.39	0.40	ŷ = 0,366 x IC + 506.727	0.96	1,268,829
1,868	0.64	0.52	0.62	0.50	0.59	0.76	[0,63 ; 0,65] (108,0)	0.61	0.36	ŷ = 0,459 x IC + 1.273.255	0.93	2,061,636
3,317	0.63	0.12	0.62	0.41	0.67	0.80	[0,62 ; 0,64] (311,4)	0.12	0.42	ŷ = 0,226 x IC + 3.122.859	0.40	81,411,375
4,761	0.62	0.37	0.61	0.31	0.59	0.88	[0,61 ; 0,63] (625,2)	0.11	0.49	ŷ = 0,719 x IC - 1.813.111	0.75	6,434,558

n	Trailing DEPV/Invested Capital Multiples									Trailing Invested Capital Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
386	0.58	0.37	0.57	0.35	0.60	0.77	[0,57 ; 0,60] (37,9)	0.26	0.47	ŷ = 0,869 x IC - 21.624	0.98	220,878
617	0.63	0.52	0.64	0.50	0.67	0.78	[0,62 ; 0,63] (40,3)	-0.36	0.32	ŷ = 0,755 x IC - 118.301	0.97	646,612
209	0.57	0.34	0.56	0.30	0.54	0.75	[0,54 ; 0,60] (24,5)	0.30	0.55	ŷ = 0,910 x IC - 17.144	0.96	59,512
945	0.70	0.53	0.71	0.51	0.75	0.88	[0,69 ; 0,71] (92,8)	-0.29	0.38	ŷ = 0,739 x IC + 180.044	0.96	1,456,938
177	0.47	0.18	0.45	0.27	0.40	0.65	[0,44 ; 0,49] (19,5)	0.56	0.61	ŷ = 0,219 x IC + 53.961	0.89	113,037
408	0.71	0.46	0.73	0.51	0.71	1.03	[0,69 ; 0,73] (55,2)	-0.23	0.45	ŷ = 1,082 x IC - 65.956	0.98	315,622
843	0.40	0.26	0.35	0.18	0.27	0.52	[0,39 ; 0,41] (79,3)	1.31	0.74	ŷ = 0,189 x IC + 379.494	0.69	1,534,107
832	0.65	0.37	0.66	0.52	0.67	0.78	[0,65 ; 0,66] (39,0)	-0.11	0.37	ŷ = 0,784 x IC - 135.136	0.92	2,728,129
338	0.59	0.42	0.60	0.47	0.59	0.76	[0,58 ; 0,61] (26,2)	-0.19	0.40	ŷ = 0,547 x IC + 257.077	0.89	821,524
1,197	0.54	0.27	0.53	0.25	0.44	0.93	[0,52 ; 0,56] (177,5)	0.33	0.63	ŷ = 1,045 x IC - 2.088.644	0.89	2,739,409
649	0.52	0.34	0.49	0.31	0.45	0.71	[0,50 ; 0,53] (56,3)	0.72	0.53	ŷ = 0,525 x IC + 1.438	0.92	159,216
392	0.59	0.34	0.59	0.27	0.61	0.93	[0,57 ; 0,62] (61,1)	0.01	0.55	ŷ = 0,891 x IC - 360.961	0.93	1,641,268
864	0.69	0.13	0.70	0.42	0.70	0.95	[0,68 ; 0,71] (104,9)	-0.13	0.45	ŷ = 0,935 x IC - 161.676	0.89	1,073,079
1,331	0.63	0.43	0.63	0.38	0.59	0.87	[0,62 ; 0,65] (168,7)	0.19	0.50	ŷ = 0,494 x IC + 89.106	0.88	3,896,770
386	0.49	0.26	0.46	0.22	0.34	0.79	[0,46 ; 0,52] (52,3)	0.61	0.69	ŷ = 0,097 x IC + 183.863	0.34	381,474
365	0.65	0.47	0.64	0.47	0.68	0.76	[0,63 ; 0,67] (26,5)	0.15	0.42	ŷ = 0,713 x IC - 105.689	0.96	365,709
617	0.59	0.26	0.58	0.34	0.55	0.83	[0,57 ; 0,61] (69,2)	0.23	0.55	ŷ = 0,543 x IC + 25.240	0.89	218,833
542	0.70	0.39	0.71	0.49	0.76	0.91	[0,68 ; 0,72] (58,5)	-0.38	0.43	ŷ = 0,830 x IC + 14.324	0.97	1,212,298

Southern Europe - Trailing & Forward DEPV/Sales (operating), 1 January 2020 until 31 December 2022

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
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n	Trailing DEPV/Sales (operating) Multiples									Trailing Sales (operating) Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
349	0.92	0.46	0.84	0.34	0.82	1.31	[0,80 ; 1,03] (29,6)	0.94	0.75	$\hat{y}$ = 0,227 x Sales + 31.830	0.21	58,417
386	1.13	0.61	1.02	0.47	0.73	1.73	[0,93 ; 1,32] (51,2)	0.92	0.81	$\hat{y}$ = 1,427 x Sales + 88.168	0.61	590,153
204	0.87	0.37	0.79	0.29	0.55	1.36	[0,69 ; 1,06] (21,9)	1.00	0.88	$\hat{y}$ = 2,187 x Sales - 56.656	0.89	100,083
510	1.11	0.48	1.03	0.43	0.96	1.72	[0,97 ; 1,25] (58,6)	0.75	0.75	$\hat{y}$ = 0,179 x Sales + 253.040	0.15	607,661
150	0.62	0.14	0.50	0.25	0.41	0.57	[0,45 ; 0,80] (19,4)	1.98	1.09	$\hat{y}$ = 0,225 x Sales + 46.559	0.86	103,378
392	1.11	0.46	1.04	0.49	0.90	1.63	[0,97 ; 1,25] (42,9)	0.70	0.70	$\hat{y}$ = 1,783 x Sales - 19.882	0.71	292,619
800	0.82	0.50	0.69	0.46	0.56	0.80	[0,75 ; 0,90] (87,4)	1.87	0.82	$\hat{y}$ = 0,516 x Sales + 210.751	0.88	841,975
279	1.19	0.20	1.11	0.47	1.01	1.59	[0,95 ; 1,42] (33,4)	0.73	0.79	$\hat{y}$ = 0,639 x Sales + 389.057	0.57	1,651,395
193	1.06	0.39	0.98	0.45	0.87	1.24	[0,82 ; 1,31] (18,2)	1.15	0.82	$\hat{y}$ = 0,910 x Sales + 391.295	0.84	1,117,320
612	0.86	0.44	0.74	0.38	0.65	1.12	[0,77 ; 0,95] (49,8)	1.52	0.81	$\hat{y}$ = 0,680 x Sales + 38.422	0.31	1,133,817
676	0.72	0.28	0.59	0.25	0.43	0.97	[0,64 ; 0,81] (60,4)	1.67	0.97	$\hat{y}$ = 0,242 x Sales + 121.558	0.42	447,618
193	0.74	0.53	0.69	0.39	0.62	1.01	[0,68 ; 0,80] (16,5)	0.93	0.58	$\hat{y}$ = 0,871 x Sales - 47.551	0.86	133,331
660	1.05	0.10	1.00	0.42	0.86	1.59	[0,96 ; 1,15] (77,6)	0.58	0.70	$\hat{y}$ = 1,607 x Sales - 6.372	0.94	109,371
1,299	1.08	0.45	1.02	0.44	0.92	1.71	[1,01 ; 1,15] (158,9)	0.51	0.70	$\hat{y}$ = 1,770 x Sales - 76.007	0.99	733,942
258	1.05	0.12	0.98	0.31	0.55	1.95	[0,79 ; 1,31] (36,9)	0.68	0.91	$\hat{y}$ = 0,560 x Sales - 7.298	0.46	197,901
86	1.14	0.67	1.10	0.68	0.94	1.58	[0,88 ; 1,39] (9,1)	0.62	0.63	$\hat{y}$ = 1,747 x Sales - 22.164	0.88	111,528
574	0.90	0.24	0.80	0.27	0.63	1.51	[0,78 ; 1,03] (59,7)	1.02	0.91	$\hat{y}$ = 1,825 x Sales - 20.335	0.93	150,500
343	0.85	0.32	0.77	0.33	0.70	1.19	[0,74 ; 0,96] (22,6)	1.13	0.79	$\hat{y}$ = 0,241 x Sales + 135.883	0.13	1,047,810

n	Forward DEPV/Sales (operating) Multiples									Forward Sales (operating) Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
81	0.94	0.62	0.92	0.60	0.63	1.32	[0,79 ; 1,10] (9,6)	0.70	0.59	$\hat{y}$ = 0,405 x Sales + 1.099.306	0.81	1,504,105
655	1.37	1.20	1.33	1.07	1.15	1.63	[1,33 ; 1,42] (64,6)	0.84	0.37	$\hat{y}$ = 0,897 x Sales + 1.409.385	0.75	1,351,163
64	1.35	1.01	1.28	0.88	1.17	1.44	[1,02 ; 1,67] (4,5)	1.16	0.56	$\hat{y}$ = 3,468 x Sales - 1.718.104	0.94	578,253
692	0.85	0.52	0.80	0.37	0.84	1.13	[0,81 ; 0,89] (45,6)	0.84	0.60	$\hat{y}$ = 0,280 x Sales + 3.086.652	0.86	2,881,140
43	0.46	0.34	0.46	0.38	0.49	0.57	[0,44 ; 0,48] (2,5)	-0.70	0.37	$\hat{y}$ = 0,249 x Sales + 329.440	0.43	1,167,058
274	1.07	0.69	0.98	0.41	0.75	1.57	[0,93 ; 1,20] (25,6)	0.81	0.65	$\hat{y}$ = 0,365 x Sales + 821.071	0.91	811,853
580	0.51	0.37	0.44	0.27	0.34	0.66	[0,48 ; 0,54] (148,2)	2.32	0.74	$\hat{y}$ = 0,238 x Sales + 1.425.438	0.45	1,636,936
445	0.98	0.60	0.94	0.58	1.07	1.15	[0,92 ; 1,04] (20,4)	1.19	0.54	$\hat{y}$ = 0,656 x Sales + 2.668.462	0.56	7,467,343
199	0.46	0.36	0.39	0.34	0.36	0.50	[0,41 ; 0,51] (948,4)	4.58	0.83	$\hat{y}$ = 0,680 x Sales - 1.536.851	0.25	6,988,265
682	0.75	0.16	0.61	0.18	0.26	1.24	[0,62 ; 0,88] (78,2)	1.18	1.16	$\hat{y}$ = 0,601 x Sales - 493.240	0.37	6,940,707
343	1.21	0.44	1.20	0.36	1.09	2.14	[1,03 ; 1,39] (57,2)	0.14	0.71	$\hat{y}$ = 0,172 x Sales + 2.211.932	0.06	2,660,829
177	0.62	0.29	0.47	0.22	0.35	0.51	[0,44 ; 0,81] (19,7)	1.86	1.18	$\hat{y}$ = 0,368 x Sales + 2.644.855	0.34	5,751,460
311	1.29	1.03	1.24	0.88	1.25	1.51	[1,20 ; 1,37] (19,4)	0.64	0.44	$\hat{y}$ = 2,254 x Sales - 1.011.882	0.86	1,051,963
558	1.30	0.74	1.28	0.81	1.36	1.57	[1,22 ; 1,38] (53,3)	0.13	0.50	$\hat{y}$ = 1,350 x Sales - 54.644	0.95	3,463,715
70	0.38	0.24	0.31	0.17	0.25	0.26	[0,32 ; 0,44] (6,1)	1.68	0.89	$\hat{y}$ = 0,071 x Sales + 144.898	-0.03	99,890
140	1.47	1.34	1.39	1.21	1.28	1.50	[1,37 ; 1,57] (12,1)	1.67	0.35	$\hat{y}$ = 1,407 x Sales + 15.192	0.50	268,655
161	0.66	0.43	0.57	0.39	0.52	0.64	[0,56 ; 0,76] (116,5)	2.77	0.81	$\hat{y}$ = 0,485 x Sales + 102.488	0.93	237,180
247	1.04	0.43	0.97	0.41	1.10	1.34	[0,89 ; 1,19] (18,7)	0.74	0.70	$\hat{y}$ = 0,570 x Sales + 2.517.023	0.43	6,423,431

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Southern Europe - Trailing & Forward DEPV/EBITDA, 1 January 2020 until 31 December 2022

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
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C28 - C30, C33	Manufacture of machinery, motor vehicles, other transport equipment; repair/installation
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NACE Rev. 2 Sector	
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n	Trailing DEPV/EBITDA Multiples									Trailing EBITDA Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
274	7.86	5.11	7.40	4.31	7.05	10.29	[2,05 ; 13,68] (23,0)	0.75	0.59	$\hat{y}$ = 9,691 x EBITDA - 37.654	0.98	241,588
338	8.45	5.19	8.14	3.84	4.45	14.69	[0,46 ; 16,44] (56,9)	0.47	0.67	$\hat{y}$ = 7,245 x EBITDA + 245.743	0.53	721,506
150	8.07	4.02	7.77	3.12	4.95	12.75	[-3,92 ; 20,06] (23,6)	0.38	0.71	$\hat{y}$ = 12,486 x EBITDA - 10.633	0.93	113,998
574	9.09	4.69	8.96	4.57	8.67	12.82	[4,28 ; 13,90] (72,9)	0.21	0.55	$\hat{y}$ = 11,387 x EBITDA - 57.598	0.97	984,611
150	5.74	0.97	5.19	3.08	4.73	6.50	[-1,43 ; 12,91] (11,2)	1.55	0.77	$\hat{y}$ = 8,085 x EBITDA + 6.669	0.83	397,082
311	8.23	4.21	7.97	4.60	7.34	10.98	[1,96 ; 14,49] (36,0)	0.41	0.60	$\hat{y}$ = 14,997 x EBITDA - 52.975	0.96	145,160
773	7.79	4.85	7.17	4.85	6.86	8.61	[4,21 ; 11,37] (65,3)	1.19	0.60	$\hat{y}$ = 5,793 x EBITDA + 504.143	0.78	1,140,353
424	9.54	2.79	9.58	5.21	10.06	14.23	[3,22 ; 15,85] (62,4)	-0.01	0.56	$\hat{y}$ = 8,929 x EBITDA + 1.251.654	0.79	3,651,625
193	9.82	3.92	9.88	3.50	10.29	15.74	[-2,37 ; 22,00] (31,4)	-0.16	0.62	$\hat{y}$ = 10,360 x EBITDA + 258.972	0.92	673,319
370	7.71	5.43	7.29	4.40	6.76	9.19	[3,11 ; 12,31] (31,9)	0.91	0.57	$\hat{y}$ = 4,087 x EBITDA + 487.443	0.68	1,271,290
521	7.33	3.75	6.79	3.05	5.56	9.23	[1,78 ; 12,88] (59,1)	0.89	0.72	$\hat{y}$ = 7,795 x EBITDA + 27.340	0.70	491,223
188	9.20	4.45	9.00	3.14	9.61	13.34	[-2,23 ; 20,64] (27,1)	0.15	0.64	$\hat{y}$ = 12,831 x EBITDA + 59.602	0.92	2,663,824
687	5.66	0.73	5.38	3.92	5.18	7.01	[3,89 ; 7,43] (29,3)	1.11	0.57	$\hat{y}$ = 4,960 x EBITDA + 50.902	0.82	267,609
1,261	7.16	3.66	6.54	3.04	5.61	9.80	[3,68 ; 10,65] (134,8)	0.96	0.73	$\hat{y}$ = 6,068 x EBITDA + 181.669	0.81	4,202,867
274	8.14	3.43	7.64	2.70	7.72	12.36	[-1,22 ; 17,51] (34,0)	0.53	0.72	$\hat{y}$ = 10,858 x EBITDA - 101.473	0.63	477,585
129	8.91	5.30	8.68	5.32	9.37	12.35	[-0,15 ; 17,96] (10,7)	0.17	0.53	$\hat{y}$ = 8,332 x EBITDA + 38.544	0.64	215,168
451	5.70	2.43	4.92	2.48	4.34	7.15	[0,75 ; 10,65] (36,3)	1.35	0.84	$\hat{y}$ = 12,922 x EBITDA - 17.655	0.90	166,162
333	6.88	2.70	6.33	3.06	5.75	9.38	[0,77 ; 12,98] (32,1)	0.87	0.72	$\hat{y}$ = 5,780 x EBITDA + 289.274	0.62	1,753,034

n	Forward DEPV/EBITDA Multiples									Forward EBITDA Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
81	5.36	4.40	5.19	3.30	4.84	7.12	[2,44 ; 8,28] (6,8)	0.74	0.45	$\hat{y}$ = 3,558 x EBITDA + 475.463	0.89	1,137,363
655	5.97	5.68	5.84	5.18	5.52	6.59	[5,62 ; 6,31] (33,6)	1.15	0.23	$\hat{y}$ = 4,864 x EBITDA + 634.883	0.90	900,662
134	9.53	6.79	9.54	4.90	11.12	13.50	[1,72 ; 17,33] (21,4)	-0.15	0.47	$\hat{y}$ = 15,882 x EBITDA - 1.793.160	0.91	998,102
741	5.08	3.59	4.61	2.48	4.64	5.77	[3,42 ; 6,74] (50,9)	1.44	0.62	$\hat{y}$ = 1,805 x EBITDA + 3.102.355	0.87	2,655,170
43	3.16	2.17	3.16	1.73	3.50	4.36	[1,48 ; 4,83] (6,9)	-0.17	0.49	$\hat{y}$ = 2,137 x EBITDA + 284.339	0.37	1,227,711
392	6.95	5.61	6.55	4.56	6.34	8.60	[4,48 ; 9,42] (28,4)	0.98	0.47	$\hat{y}$ = 4,806 x EBITDA + 894.081	0.66	1,620,054
580	3.64	2.96	3.19	2.29	2.92	4.28	[2,75 ; 4,53] (280,5)	2.62	0.60	$\hat{y}$ = 2,088 x EBITDA + 1.179.077	0.47	1,603,574
617	5.42	4.65	5.29	4.15	5.24	6.26	[4,67 ; 6,17] (46,7)	0.65	0.37	$\hat{y}$ = 3,194 x EBITDA + 3.384.530	0.75	4,846,643
193	3.21	2.64	3.14	2.05	3.17	4.29	[2,65 ; 3,77] (11,8)	0.49	0.41	$\hat{y}$ = 3,755 x EBITDA - 713.341	0.71	1,340,818
794	5.54	2.16	4.76	2.03	3.92	7.61	[2,04 ; 9,05] (59,9)	1.37	0.84	$\hat{y}$ = 4,402 x EBITDA + 750.741	0.71	5,004,398
338	5.67	4.55	5.60	3.68	5.74	7.32	[4,30 ; 7,03] (38,8)	0.26	0.41	$\hat{y}$ = 5,756 x EBITDA - 136.718	0.87	1,123,625
188	7.28	4.11	6.75	2.73	5.84	11.74	[-1,93 ; 16,49] (21,7)	0.77	0.72	$\hat{y}$ = 2,070 x EBITDA + 7.718.531	0.10	9,496,679
349	5.19	4.11	5.04	3.33	4.84	7.32	[3,84 ; 6,54] (42,3)	0.38	0.45	$\hat{y}$ = 4,885 x EBITDA - 41.813	0.64	4,514,120
660	5.46	3.77	5.03	3.69	4.38	6.25	[3,56 ; 7,36] (48,5)	1.38	0.60	$\hat{y}$ = 3,992 x EBITDA + 851.012	0.91	4,545,529
11	-	-	-	-	-	-	-	-	-	-	-	-
172	11.44	9.74	11.13	7.44	9.91	15.39	[3,79 ; 19,08] (25,0)	0.57	0.41	$\hat{y}$ = 19,539 x EBITDA - 758.624	0.94	577,642
145	4.81	3.19	4.55	3.72	4.32	4.98	[1,67 ; 7,96] (22,4)	1.81	0.60	$\hat{y}$ = 4,348 x EBITDA + 63.686	0.94	217,916
370	6.44	4.59	6.05	3.84	5.04	8.83	[3,32 ; 9,56] (30,8)	0.94	0.56	$\hat{y}$ = 4,083 x EBITDA + 1.504.551	0.89	2,482,092

Southern Europe - Trailing & Forward DEPV/EBIT, 1 January 2020 until 31 December 2022

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
C19 - C23	Manufacture of coke, chemicals, rubber, refined petroleum/chemical/pharmaceutical/plastic/mineral products
C24 - C25	Manufacture of basic metals, fabricated metal products
C26 - C27	Manufacture of computers, electronic/optical products, electrical equipment
C28 - C30, C33	Manufacture of machinery, motor vehicles, other transport equipment; repair/installation
D35	Electricity, gas, steam and air conditioning supply
E36 - E39	Water supply, sewerage, waste management, remediation activities
F41 - F43	Construction - Buildings, civil engineering, specialized construction activities
G45 - G47	Wholesale/Retail trade, repair of motor vehicles and motorcycles
H49 - H53	Transportation and storage - Land/pipelines, water, air; warehousing, postal/courier activities
J58 - J60, C18	Publishing activities, programme production, music publishing, broadcasting, printing
J61 - J63	Telecommunications, computer programming/consultancy, information service activities
K64 - K66	Financial and insurance activities
L68	Real estate activities
M69, M70, M73, N77 - N82	Legal/accounting activities, consultancy, advertising/market research, rental/employment/security activities, travel agency
M71, M72, M74, M75	Architectural/engineering/other professional activities, technical testing, scientific R&D, veterinary activities

NACE Rev. 2 Sector	
C10 - C12	Manufacture of food products, beverages, tobacco products
C13 - C15	Manufacture of textiles, wearing apparel, teather and related products
C16, C17, C31, C32	Manufacture of wood/products, paper/products, furniture; other manufacturing
C19 - C23	Manufacture of coke, chemicals, rubber, refined petroleum/chemical/pharmaceutical/plastic/mineral products
C24 - C25	Manufacture of basic metals, fabricated metal products
C26 - C27	Manufacture of computers, electronic/optical products, electrical equipment
C28 - C30, C33	Manufacture of machinery, motor vehicles, other transport equipment; repair/installation
D35	Electricity, gas, steam and air conditioning supply
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M71, M72, M74, M75	Architectural/engineering/other professional activities, technical testing, scientific R&D, veterinary activities

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n	Trailing DEPV/EBIT Multiples									Trailing EBIT Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
231	11.98	6.87	11.40	5.44	10.02	17.59	[-5,36 ; 29,31] (28,8)	0.48	0.64	ŷ = 10,972 x EBIT + 77.913	0.84	1,052,866
301	12.00	7.70	11.46	5.98	9.05	18.89	[-2,50 ; 26,51] (39,8)	0.55	0.62	ŷ = 17,817 x EBIT - 43.089	0.90	301,572
129	10.77	6.26	10.03	4.57	9.23	14.33	[-10,25 ; 31,79] (11,2)	0.82	0.67	ŷ = 12,274 x EBIT + 24.019	0.80	195,723
564	12.49	5.98	12.08	7.06	11.45	17.06	[3,06 ; 21,93] (58,3)	0.48	0.56	ŷ = 11,927 x EBIT + 225.033	0.95	1,403,818
134	9.34	1.19	8.66	4.68	6.93	14.13	[-8,24 ; 26,91] (6,9)	1.13	0.72	ŷ = 8,116 x EBIT + 36.619	0.83	412,839
274	11.12	4.92	10.21	4.69	9.82	17.46	[-6,33 ; 28,57] (26,8)	0.81	0.72	ŷ = 19,619 x EBIT - 45.997	0.92	233,822
698	14.32	6.93	14.23	9.05	14.71	19.34	[5,28 ; 23,36] (69,6)	0.01	0.51	ŷ = 13,045 x EBIT + 526.802	0.83	982,186
392	12.78	3.60	12.44	7.26	13.07	17.14	[0,61 ; 24,94] (38,0)	0.37	0.57	ŷ = 11,364 x EBIT + 1.979.940	0.69	6,315,000
145	12.88	4.98	12.55	4.69	10.60	19.15	[-18,80 ; 44,56] (18,7)	0.45	0.71	ŷ = 9,582 x EBIT + 219.875	0.96	486,801
413	11.70	6.59	11.21	6.66	8.94	17.47	[0,31 ; 23,08] (42,0)	0.68	0.61	ŷ = 13,278 x EBIT - 102.443	0.63	5,995,620
472	10.18	4.56	9.30	4.19	6.86	16.27	[-2,31 ; 22,68] (49,2)	0.89	0.76	ŷ = 13,084 x EBIT - 9.887	0.73	691,885
209	13.76	7.11	13.52	8.28	13.82	19.89	[-5,01 ; 32,53] (23,0)	0.15	0.56	ŷ = 17,874 x EBIT + 67.251	0.87	3,158,067
633	7.91	0.80	7.50	4.23	7.47	11.08	[3,60 ; 12,22] (42,6)	0.78	0.62	ŷ = 6,087 x EBIT + 80.345	0.74	326,366
1,079	10.35	4.33	9.50	3.76	7.47	16.25	[1,77 ; 18,94] (127,6)	0.73	0.76	ŷ = 11,785 x EBIT - 31.599	0.99	774,155
360	12.96	5.36	12.65	5.43	10.63	22.00	[-4,38 ; 30,29] (52,7)	0.34	0.66	ŷ = 18,556 x EBIT - 90.565	0.77	539,719
150	16.36	9.95	16.43	9.70	13.96	24.31	[-11,98 ; 44,70] (21,9)	0.08	0.53	ŷ = 28,656 x EBIT - 219.059	0.97	392,067
445	8.65	2.29	7.63	2.79	5.72	13.05	[-3,84 ; 21,15] (42,7)	1.07	0.88	ŷ = 14,109 x EBIT + 10.393	0.84	228,921
354	9.91	3.06	9.03	4.09	6.54	15.96	[-5,76 ; 25,57] (39,4)	0.89	0.81	ŷ = 6,540 x EBIT + 473.581	0.55	2,249,037

n	Forward DEPV/EBIT Multiples									Forward EBIT Regression		
	$\bar{x}_a$	$\bar{x}_h$	$\bar{x}_t$	Q_1	Q_2	Q_3	95% (JB)	sk	cv	$\hat{y}$ = DEPV (TEUR)	$\bar{R}^2$	se_y
81	8.68	7.45	8.19	5.82	8.74	10.04	[1,74 ; 15,63] (4,2)	1.30	0.43	ŷ = 5,805 x EBIT + 443.723	0.87	1,237,335
655	10.35	9.07	9.79	7.68	8.19	12.44	[7,12 ; 13,58] (49,7)	1.32	0.41	ŷ = 6,646 x EBIT + 1.037.643	0.91	851,774
134	13.83	9.89	13.90	7.84	14.17	19.16	[-0,64 ; 28,30] (19,1)	-0.16	0.44	ŷ = 20,653 x EBIT - 1.202.123	0.87	1,185,671
741	7.91	5.37	7.27	3.54	7.24	8.90	[3,73 ; 12,10] (58,6)	1.12	0.63	ŷ = 2,577 x EBIT + 3.164.625	0.87	2,608,429
43	4.51	3.14	4.51	2.30	5.40	6.09	[1,32 ; 7,71] (7,6)	-0.34	0.48	ŷ = 4,394 x EBIT - 65.956	0.66	902,072
392	10.61	8.17	10.27	6.46	9.69	15.41	[4,37 ; 16,84] (42,0)	0.60	0.49	ŷ = 6,659 x EBIT + 957.222	0.68	1,568,920
569	5.57	4.36	4.72	3.30	4.23	6.43	[2,46 ; 8,68] (638,7)	3.27	0.73	ŷ = 3,214 x EBIT + 872.993	0.70	1,049,803
617	8.53	7.11	8.18	6.52	8.10	9.71	[6,16 ; 10,89] (52,0)	1.50	0.42	ŷ = 5,156 x EBIT + 3.324.341	0.75	4,809,294
193	5.62	4.02	5.61	2.61	5.77	7.84	[3,14 ; 8,10] (27,6)	0.08	0.49	ŷ = 3,491 x EBIT + 1.227.480	0.23	2,172,454
762	9.40	4.15	8.46	3.29	7.67	13.47	[1,09 ; 17,71] (60,2)	1.03	0.76	ŷ = 6,991 x EBIT + 956.020	0.78	4,015,532
301	8.67	6.99	8.70	6.88	7.81	11.46	[5,86 ; 11,48] (28,2)	0.08	0.38	ŷ = 7,846 x EBIT + 3.747	0.89	1,057,003
188	13.03	8.11	12.54	6.10	14.65	17.95	[-6,21 ; 32,28] (21,1)	0.40	0.58	ŷ = 5,939 x EBIT + 5.359.336	0.24	8,749,055
317	10.30	8.12	9.67	6.15	8.42	13.51	[3,09 ; 17,51] (22,3)	1.11	0.52	ŷ = 16,522 x EBIT - 937.340	0.71	1,702,742
606	11.82	8.71	11.55	6.49	12.47	15.13	[6,24 ; 17,40] (48,5)	0.36	0.47	ŷ = 13,394 x EBIT - 64.594	0.94	3,539,894
11	-	-	-	-	-	-	-	-	-	-	-	-
188	12.15	8.93	11.79	7.70	9.87	17.41	[1,07 ; 23,23] (19,1)	0.61	0.48	ŷ = 18,426 x EBIT - 564.917	0.93	577,110
134	7.40	5.83	7.15	5.10	6.44	7.15	[2,02 ; 12,79] (10,7)	1.06	0.50	ŷ = 5,922 x EBIT + 116.043	0.93	240,939
370	9.17	6.87	8.82	6.52	8.86	11.29	[4,63 ; 13,70] (24,6)	0.79	0.48	ŷ = 6,598 x EBIT + 1.178.791	0.91	2,140,381

News from IVSC

The IVSC has a bold new look!

The IVSC has recently unveled a refreshed brand including updated logo. This is the first such update to the organisation's brand since its inception in the 1980s. It comes off the back of a consutation with member and sponsor organisations who provided input to the design process. The IVSC's new logo places an emphasis on the importance of valuation. The new 'V' design highlights the central role of valuaiton in the organisations work and recognises the importance of IVSC in driving up standards and confidence in valuations worldwide. Accompanying the new branding, the IVSC has also created brand use guidelines and logos that can be used by its member and sponsor organisations to promote their involvement with the organisation.

Watch: Building Trust in Valuation: https://vimeo.com/821713309.

The International Valuation Standards are changing – have your say!

IVSC has launched a three-month consultation on proposed updates to the International Valuation Standards (IVS). The consultation, which runs until 28 July 2023, is part of the IVSC's two-yearly cycle of standards updates. IVSC is seeking feedback from all interested parties to ensure the standards evolve reflecting practitioner and market needs, and international best practices.

Key changes proposed in the consultation Exposure Draft include:

- Simplifying the standards and making them more relevant for the multitude of participants in the valuation process.
- Introducing new features to enhance the user experience, such as hover-over definitions, intuitive navigation, and asset-specific guidance.
- A full update to IVS 500 - Financial Instruments
- Adding new chapters on Data and Inputs (IVS 104) and Valuation Models (IVS 105).
- Updating mandatory elements and expanding the scope of work requirements, including Environmental, Social, and Governance (ESG) considerations.

The consultation documents and feedback form are available on the IVSC website. You can submit your responses using the online form or via email.
Watch again - Webinar on key changes to the IVS: https://vimeo.com/827318551.

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EY Global Valuation Leader, Adrian Nicholls, on emerging trends in valuation

As part of its regular series of interviews with valuation leaders, the IVSC has recently published an interview with EY Global Valuation Leader, Adrian Nicholls. In it, Adrian discusses some of the trends that are fundamentally changing the world of valuation and discusses his career within the profession. EY is a Sponsor of the IVSC and has adopted the International Valuation Standards as a framework across its global valuation business.

You can download Adrian's interview form the IVSC website: www.ivsc.org/charting-the-future-of-valuation-reflections-from-eys-global-valuation-leader.

IVSC AGM – Paris, 9–11 October 2023

The IVSC's 2023 AGM will take place in Paris from 9-11 October. Kindly sponsored and hosted by Conseil National de l'Ordre des Experts-Comptables (CNOEC) and Compagnie Nationale des Commissaires aux Comptes (CNCC), the AGM programme will include a one-day valuation conference on 10 October, exploring a range of international and regional valuation topics through workshops and plenary sessions. The AGM provides an opportunity for the IVSC's boards and member organisations to catch up in person and to reflect on developments across the profession. Further details on the programme and how you can participate can be found on the IVSC website.

New Perspectives Paper: Deciphering Technology

The introduction of new technology often marks the beginning of a new era: railroads, electrification, and combustion engines produced momentous changes even before the advent of the "digital revolution". The current wave of innovation is one of the factors behind the rise of intangible assets, which now account for a larger proportion of corporate assets than tangible ones.

This transformation towards more intangible assets has had profound effects on the valuation of assets and businesses. It is the object of the current series of Perspective Papers the IVSC has published. In Parts 1 and 2 of our series, we examined the "Case for Realigning Reporting Standards with Modern Value Creation" and focused on human capital. In Part 3, we examined brands and reputation. In this paper, the fourth of our series, we address the topic of technology valuation. You can download the paper here.

News from EACVA

New Section Coming Soon: EBVM's Spotlight on Country-Specific Valuation Topics

EBVM is a practice-oriented and scientific-based magazine. We welcome both practical and scientific articles to share their thoughts on business valuation topics also with a country specific view.

We are very interested in receiving articles on **country-specific valuation issues from around the globe**. There is a lot that you can tell the world:

- What should others know about business valuation in your country?
- What approaches (income, market, cost) are used in your region and for what purpose?
- What are the main valuation purposes and bases of value (standards of value) used in your country?
- Is there a favourite DCF-technique used by practitioners in your country (WACC, APV, FtE etc.)?
- What are the common terminal value assumptions?
- Forecasting or projection: What is usually the basis for valuation in your country?
- How important is the use of transaction prices or stock prices in your region?
- Do valuators in your region refer to country-specific valuation standards and/or international standards or best practices in your country?
- What data sources are available in your country? Is there something available for private companies?
- Do you see any economic or industry topics which relate to your region and are worth to talk about?
- What happens in court? Is there any case law / court decisions based on the valuation purposes (transaction, family law, shareholder disputes, tax, accounting etc.) in your country?

Please let the world know about your country-specific challenges and solutions in business valuation: EBVM@eacva.de.

Certified Valuation Analyst (CVA)

– Cost of Capital Parameters for the Case Study –

As the largest business valuation association in Europe, EACVA membership comes with many benefits, such as unique networking opportunities, subscriptions to renowned journals, valuable information sources and access to exclusive events. Most importantly, however, the EACVA passionately follows their vision *to share and distribute knowledge*. As a result, the EACVA has trained over 1,400 individuals in Europe since 2005 to become globally recognised as a Certified Valuation Analyst (CVA). To further support EACVA members in their goal to excel in the business valuation profession, KPMG now provides CVA candidates with access to the complete range of features of the KPMG Valuation Data Source – for a period of 4 weeks.

The data source provides access to cost of capital parameters from more than 150 countries and sector- as well as peer-group-specific data from over 16,500 companies worldwide. Valuation experts therefore have access to reliable and quality-assured data at the push of a button. More information and access to the free trial version here: www.kpmg.de/en/valuation-data-source

EACVA's Live Web Seminar: Start-Up Valuation

– Analysis and Valuation of Young and Innovative Business Models –

EACVA invites you to attend our live web seminar **15 November 2023** on valuation of start-ups and young innovative firms, in which you will get familiar with necessary analytical steps, common techniques and approaches to the valuation of start-up companies and innovative business models. Several *practical examples and cases* complement the presentation.

After completing this web seminar, you will be able to apply proper fundamental analysis techniques to different start-up cases, to approach the valuation problem by use of appropriate techniques and to deal with the often huge uncertainty that exists in early stage business models.

Content:
- Basics of analysing start-up companies
- Specific challenges in understanding young business models (drivers, failures)
- Approaching data-driven businesses
- Valuation techniques, approaches and special cases
- Typical pitfalls in valuing start-up companies

» Learn more and register...

EACVA's 16th Annual International Business Valuation Conference

30 November and 1 December 2023 I Hotel Palace Berlin

We cordially invite you to attend our 16th International Annual Conference for Valuation Professionals at the Hotel Palace Berlin, located in the middle of the City West. It will provide an exciting learning opportunity for all attendees to hear from some of the most renowned speakers in the business valuation field while connecting and networking with other valuation professionals.

Highlights:
- Two conference days with 32 renowned speakers
- A varied programme with 25 sessions, keynotes and panel discussions on current issues in business valuation
- Keynote sessions: Dealing with Uncertainty in the Digital Age (Prof. Dr. Gerd Gigerenzer) / Matching Risk and Return: Observations on Developing Discount Rates (Roger J. Grabowski, FASA)
- Networking dinner and the magic show at the Wintergarten Varieté Berlin (*complimentary for all conference participants*)
- Exciting learning & netwrking opportunities *and much more...*

Save the Date! More information coming soon on www.ValuationConference.de

IVSC Members Introduce Themselves:

ANEVAR, the National Association of Authorized Romanian Valuers, was established in 1992 and enhanced its role in 2011, when the valuation profession became regulated by the Government Ordinance No. 24/2011 on asset valuation, as approved with amendments by Law No. 99/2013. As a result, the valuation activity in Romania is supervised by ANEVAR and can only be performed by authorised valuers who acquire this qualification in line with the above-mentioned legal act.

How would you describe your organisation?

ANEVAR is the sole valuers organisation in Romania and includes more than 4,000 authorized individual valuers and over 600 authorized companies. Taking into account these numbers ANEVAR is among the largest valuer professional associations in Europe. Our organisation is now a mature professional association with recognised public utility. The authorised valuers, members of ANEVAR mainly perform the following activities: Business valuation, including valution of goodwill and other intangible assets, valuation of real estate, valuation of movable assets, valuation of shares and other financial instruments, valuation Review.

ANEVAR is well-known and appreciated in our country and abroad for the quality of the training courses and continuous professional education which consist of conferences, seminars and other activities.

Thanks to more than 30 years of constant evolution of the valuer profession in Romania coordinated by our association, we are proud to support organisations from other countries to develop strong and healthy professional associations of valuers. Our trainers are delivering training courses, seminars and participate to various conferences and international working groups in order to develop profession.

One of the most important assets of our organisation is the way it is lead by a Board of Directors which includes the actual President, the Elected President (which will be the next President after 2 years), the Former President and other 8 individuals. This is the way to ensure the continuity of important projects.

Please tell us about your member structure!

According to the law, the valuation services can only be delivered in our country by authorised valuers who are registered in the Valuers' Register of ANEVAR. It includes the following categories of authorized members:

- Full members (individuals);
- Corporate members;
- Trainee members (natural persons);
- Accredited members (natural persons);
- Inactive members;
- Honorary members.

What are the most challenging valuation topics for your members right now?

- Valuation under uncertainty – crisis environment / a non transparent market
- Use of technology to improve the quality of valuation
- Valuation of complex financial instruments/assets, e.g. derivatives, preferred shares, corporate incentive plans
- Reflecting ESG factors within valuations (business, real estate and other type of assets). Valuation of specialised assets in the context of EU's decarbonation plans.

What valuation standards do your members follow?

ANEVAR adopted International Valuation Standards (IVS) as a basis for national standards starting January 1, 2004. Since 2012 ANEVAR issued Romanian Valuation Standards (SEV) which include IVS standards, some of European standards and Romanian issued standards in order to harmonise valuation standards to European and Romanian laws.

Why are you VPO member with IVSC?

ANEVAR has been a member of IVSC for over 25 years and we share the same values. As a regulated institution, ANEVAR serves the public interest and supports the development of the valuation profession according to the highest ethical principles and the best professional standards. Thus, the affiliation of our organisation with an elite international regulatory body of professional standards, such as IVSC, is an additional guarantee for the quality of Romanian valuers' activity. ◆

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The European Business Valuation Magazine

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